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SYNTHETIC PRESENTATION

Since my Doctorat d'État Thesis in 1980, my scientific activities are directed towards the study of *extreme phenomena* in systems governed by nonlinear partial differential equations, which is the case of many systems arising in Physics, Biology and even in Economy. I devoted an important part of these researches in the fine study of the most concentrated object which exists, the punctual singularity or blow-up localized in a single point, which is, in some sort, the indivisible atom of these extreme phenomena. At the beginning these researches might have appeared academic, and, often, I have shifted from one subject to another, among the same corpus of problems. In 1992, E.B. Dynkin contacted me and emphasized the interest that probabilists, involved in the study of superdiffusion processes, put on the works I developed with my Doctorat d'État student A. Gmira, on boundary singularities. The most elementary constituent of these singularities is the pointwise boundary singularity. Thus, I developed a scientific collaboration with Moshe Marcus, a Technion Institute Professor. From this collaboration, gradually, came out an ambitious research program centered on the notions of *boundary trace*, and *initial trace*.

What is behind the notion of boundary trace? If we suppose that some complex phenomenon occurs into an inaccessible medium in the space and that only the boundary of this medium is reachable by the observation, we shall only receive an information coming from this boundary. What will be this information? Will it be sufficient to reconstruct in a unique way the phenomenon? These are the two questions to which is associated the boundary trace theory. The initial trace deals with a phenomenon evolving in the time, and the question is to know its initial state and to see if the knowledge of this initial state is enough to characterize it in full. In our case the phenomena are governed by nonlinear partial nonlinear equations, and since there is no upper limit on the magnitude of the values it can reach, the traces are naturally very singular objects, with infinite values in many places (think for analogy with the initial blow-up in the Big-Bang theory). More precisely they are generalized Borel measures in the mathematical terminology. The boundary isolated singularities are, in some way, the microscope, thanks to what an important part of this program has been reached, in the so-called *subcritical case*, or with moderate absorption. The *supercritical case* exhibits a variety of severe difficulties, not only technical (since the above mentioned microscope no longer exists), but also conceptual. The works that I am carrying on with M. Marcus has allowed us to replace with a lot of efforts, the "point-wise microscope" and to obtain quasi-representation for some explosive solutions, under the form of infinite series involving the capacity of the blowing-up sets. These formula extends those found and used by N. Wiener for the fine study of harmonic functions. The second difficulty that we encountered lead us to replace, in the description of the boundary trace, the usual topology by a much finer one. The most recent results we have proved should rapidly imply a complete resolution of the problem of the representation of the solutions by mean of their precise trace.

Notice that recent mathematical events shows how an important tool is the fine analysis of solutions of nonlinear partial differential equations for understanding the topological structure of geometric objects, allowing G. Perelman to solve the Poincaré conjecture in dimension 3 via the study of the Ricci flow.