Mister President of the University,

Mister the Mayor of Tours

Mister Dean of the Faculty of Sciences,

Dear Colleagues and friends,

We are gathered here to honor a great mathematician, Professor James SERRIN, however the three flags on the dais must remind us that today, on the sixth of June, we also celebrate the sixty first anniversary of the day when so many young American men gave their life in Normandy to free France and Europe from the Nazi barbary. I know that James Serrin is proud of the sacrifice borne by his fellow citizens at that crucial moment of History.

Today, I have the greatest pleasure to present some facets of the personality and of the work of James SERRIN. In 1972, in my first steps in scientific research under the supervision of Professor Haïm BREZIS, who honors us with his presence today, I knew nothing about James SERRIN: my mathematical horizon at the time had been implicitly limited to my masters, Laurent SCHWARTZ, Paul MALLIAVIN, Jean LERAY, Yvonne CHOQUET-BRUHAT, Jacques-Louis LIONS among others. I had some vague idea that there existed active first rank mathematicians beyond the Atlantic or the Oder river, although I had had no encounter with their achievements in my own work.

In 1977, it was proposed to me to study the following: "*Is it possible to characterize the solutions of a semilinear elliptic equation which admits an isolated singularity?*" In the course of my research on the problem, I discovered in the bibliography that in 1964 a certain J. SERRIN had already published a paper entitled "*Local behavior of solutions of quasi-linear elliptic equations*";, in the famous journal *Acta Mathematica*. This article was followed a year later, in the same journal, by "*Isolated singularities of quasi-linear equations*". That was my first research encounter with James SERRIN.

These papers were extraordinarily rich for me: working with general operators, the p-Laplacian being a very typical example, the author, with a judicious choice of test functions, succeeded in deriving the Harnack inequalities which lead not only to the HÖLDER regularity, but also to the full characterization of the isolated singularities of solutions of such equations. From then on, I realized that "I had made acquaintance" with a giant in nonlinear analysis from Minneapolis, who had furthermore answered many of the questions I could have imagined. I met Jim in 1984 at a congress in Berkeley, and I had the opportunity of further substantial exchanges a few weeks later in Minneapolis.

What exquisite hosts Barbara and Jim SERRIN are! In their house, with antiques and memories, paintings and books, I had the privilege to have lunch on beautiful Wedgwood china, before attending a concert in their company. This completes my concise presentation of the Man, before I speak of the amplitude of James SERRIN's scientific work, its many faces, its rigor and elegance.

James SERRIN did his PhD at Indiana University in Bloomington, under the supervision of David GILBARG, a great analyst and a well known specialist in hydrodynamics. His research subject was the study of a free boundary problem in hydrodynamics. His first work was about the theory of planar wakes. Instead of considering the classical HELMOLTZ model with two free lines of a wake without intersection, James SERRIN conceived a scheme where the two lines are meeting at finite distance.

Although some of the configurations had already been studied by LAVRENTIEV, using variational methods, the method used by SERRIN was based on the LERAY-SCHAUDER degree: starting from an existing configuration, the goal is to establish *a priori* estimates and then to derive the existence of such configurations by a fixed-point theorem. Jean LERAY and Mikhail LAVRENTIEV two famous scholars, one French, the other Russian, can be considered as the precursors of James SERRIN. The very first issue of the newly published *Journal for Rational Mechanics and Analysis*, which became *Archives for Rational Mechanics and Analysis* was devoted to his thesis.

At Indiana University, he met Eberhard HOPF, known as the author of the HOPF maximum principle, but also as an astronomer, a hydrodynamicist and a specialist of ergodic theory. This encounter may explain the key role maximum principles were to play in his work. He also met with Clifford TRUESDELL, a well known specialist of mechanics in all its branches, from thermodynamics to gas dynamics, who remained eleven years in Bloomington before joining Johns Hopkins University in Baltimore. This was the beginning of a long friendship, together with James SERRIN's interest for thermodynamics, a branch of physics up to then disregarded by mathematicians.

Alongside his works in hydrodynamics, James SERRIN undertook to study the local behavior of solutions of linear elliptic equations. In 1955 he published two consecutive articles in the *Journal d'Analyse Mathématique*: the ffirst dealing with HARNACK's inequalities for linear elliptic equations, and a subsequent on with isolated singularities. Those two papers announced some of his main contributions to nonlinear analysis.

Here is another subject for reflexion and study: the calculus of variations, nowadays called variational methods. One of the questions Jim addressed and solved was the following: "*Under which conditions a function of two variables, critical point of a double integral of calculus of variations in an irregular domain of the plane, achieves a local minimum of the integral functional?*"

For the first time he defined the value of the functional by using some ideas from the theory of surfaces, a set of ideas which will come back triumphantly a few years later. The period 1963-1964 highlights the full maturity of Jim SERRIN's thought, in particular in mastering the art of well adapted test functions for studying general quasilinear equations.

It is during that period that his two articles on isolated singularities were published in *Acta Mathematica*. Shortly afterwards, in collaboration with Howard JENKINS, James SERRIN published a surprising article on construction of minimal surfaces. And this was the first evidence of his growing interest in differential geometry.

Starting from the example of the SCHERK surface, known since 1835, of a minimal graph over a square, taking the value plus infinity on two opposite sides and minus infinity on the remaining sides, the authors established necessary and sufficient conditions for constructing a minimal surface over a two-dimensional convex polygonal domain, taking a prescribed value on any side of the boundary, which is either a continuous real valued function, or plus infinity, or minus infinity.

The example of the convex quadrilateral is striking: a minimal graph taking value plus infinity on two opposite sides and a continuous value on the remaining sides does exist if and only if the length L of the sides where the value is plus infinity is smaller than the length M of the sides where the value is finite. If the finite value is replaced by minus infinity (as for the SCHERK surface), the necessary and sufficient condition turns out to be $L = M$.

This result is the first of a series where the conditions which allow to solve the DIRICHLET problem for non uniformly elliptic partial differential equations was to be brought into light. The *climax* of this work is his famous article "*The problem of Dirichlet for quasilinear differentials equations with many independent variables*", published in London in 1969 in the *Philosophical Transactions of the Royal Society*.

The description of this major contribution would be too technical, but roughly speaking, the existence of a solution of the equation is reduced to obtaining an *a priori* estimate. This *a priori* estimate is achieved by constructing global barrier functions, and finally, these barrier functions are obtained under assumptions which link the geometry of the boundary (the curvature) to the coefficients of the operator. For example, in the case of a flat graph with a prescribed mean curvature H, the DIRICHLET problem is solvable for any boundary data, if and only if the curvature k of the boundary is larger than twice the absolute value of the curvature H. Magnificent!

Another seminal article at the origin of an impressively wide lineage of mathematical publications is his paper "*A symmetry problem in potential theory*". James SERRIN makes use here, for the first time in analysis, of the combination

of the reflection and moving planes methods, introduced by the great Russian geometer Alexandroff in1945, in order to prove that if a domain admits a function with constant Laplacian, vanishing on the boundary, with a non zero constant normal derivative, then the domain must be a ball, and the function radial.

Mathematical analysis, calculus of variations and geometry were not sufficient for James Serrin. Hence, after some brilliant first contributions to hydrodynamics, he turned his research towards other problems arising from physics, such as boundary layers phenomenon, tornadoes, phase transitions and finally on thermodynamics. One of the main aspects of his work is that it is achieved through a thorough reflexion about the nature of the constitutive relations in matter and on their restrictions. He immersed himself into the study of the works of Carnot, Kelvin, Clausius, Gibbs and Caratheodory in order to present the main concepts of thermodynamics within the framework of rigorous mathematical analysis. I am not an expert in this fields, but James Serrin is now widely considered as the Bourbaki of thermodynamics.

At the end of the seventies, James Serrin returned to partial differential equations and ordinary differential equations. The fashion was then to prove existence and non-existence result following Pohozaev's spectacular method. The concept, coming from physics, of ground states which are positive solutions going to zero at infinity, appeared. The existence was obtained either by shooting methods, or by variational methods.

Much more difficult to obtain was the result of uniqueness of such solutions that he proved with B. Peletier in 1983 and 1986. The non-existence results of solutions of such or such equation plays an important role beginning with Holmgren's work at the beginning of the last century. The significance of this approach was put into light by Gidas and Spruck in a well known article from 1981. These non-existence results are usually consequences of some integral identities that are verified by the solutions of some problems.

In 1986, Patrizia Pucci and James Serrin published in the *Indiana Uni-*

versity Mathematics Journal an article which will serve as a tool box to any one looking for integral identities extending POHOZAEV's paper "*A general variational identity*".

The very powerful results contained in this work were a magnificent example of a thorough understanding of Noether's general invariant principle and its adaptation to the framework of partial differential equations.

The exemplary collaboration between Patrizia PUCCI and James SERRIN gave birth to thirty five publications on subjects as varied as the structure of the set of the critical points which are obtained by the mountain pass theorem, the asymptotic stability of solutions of second order differential equations, the maximum principles for nonlinear degenerate operators.

More recently, James SERRIN has returned to the theory of quasilinear elliptic equations which marked so much the beginning of my own research work. In collaboration with Henghui ZOU, he has obtained a spectacular result, by extending the estimates already proved by GIDAS and SPRUCK in the case of a semilinear equation to the one of a quasilinear equations: the Laplacian replaced by the p-Laplacian! The proof of GIDAS and SPRUCK's was so intricate that no one could have dreamed of adapting it to a more general situation.

This impressive work, entitled "*Cauchy-Liouville and universal boundedness theorems for quasilinear elliptic equations and inequalities*" has been published in *Acta Mathematica* in 2002. It required, among other things, a new series of regularity results, and an infinite patience of the authors, and of the referee — I can speak of it with full knowledge of the facts—. If someone has not the determination to follow all the text, I can at least advise him or her to read the introduction, a fascinating historical view which deserves the title given to this magnificent dissertation.

One must point out the exceptional quality of the journals in which James Serrin has already published his works. The Swedish journal Acta Mathematica is widely considered as one of the most prestigious mathematical publications.

James Serrin has published six articles in it over a period of thirty five years. Only the Swedish mathematician Lars Garding has done the same, and Lars Hormander, another famous Swedish mathematician, surpassed him.

After this short presentation of James SERRIN's work, I would like to say some words of his collaborators, who are all well known in the field of mathematics and mechanics. Among the main I would like to quote are David GILBARG, his PhD advisor, with whom he published works in hydrodynamics and on the singularities of solutions of elliptic equations. Robert FINN, who is here, with whom he studied the regularity of quasi conformal maps, Howard JENKINS who contributed to those important articles on minimal graphs, Donald ARONSON, who is also present, with whom he wrote an important article on the local behavior of solutions of quasilinear parabolic equations, and Hans WEINBERGER,

Bryce MCLEOD and Kevin MCLEOD, the father, with whom he published articles on fluid mechanics, and the son, who was his student, for studying uniqueness problems in semilinear equations, Roger FOSDICK, for some works in mechanics, Bert PELETIER, who is here, for nine articles on uniqueness of ground states, as well gradient estimates and LIOUVILLE type results. Some of these articles were joint work with Frederic ATKINSON, Howard LEWINE, with whom he wrote four articles on quasilinear parabolic equations, Frank MERLE, who is here, Bruno FRANCHI, Ermanno LANCONELLI, Filipo GAZZOLA, for various works treating with uniqueness of solutions of quasilinear equations which extend the previous ones obtained with B. PELETIER, Henghui ZOU and Patrizia PUCCI, who is also here, of whom I have already spoken much because of important results.

After this presentation of the major scientific advances we owe to James SERRIN, and which were published through more than one hundred and eighty articles, and the listing of his collaborators, I will not repeat the elements of his career and his *cursus honorum*. I shall restrict myself to his editing activities since he is a member of the editorial board of a dozen of mathematical journals and every one remembers the crucial role he has played as co-editor of the *Archives for Rational Mechanics and Analysis* for almost twenty five years. This journal is now considered as one of the most eminent is the field.

The Academic World has indeed recognized the quality of James SERRIN's scientific work: he received the G. D. BIRKHOFF prize in 1973 and was awarded the title of Doctor Honoris Causa from the Universities of Sussex, Ferrara and Padova. The National Academy of Sciences of the United States, and the American Academy of Art and Sciences, two most prestigious academies in America, have elected him as member. He is also a foreign member of the Academy of Sciences of Finland.

We are honoring a great scientist today, and his life perfectly fits with this verse, from Virgils Georgics, engraved on the medal commemorating this ceremony, "*Felix qui potuit rerum cognoscere causas*", that is "*Fortunate is he who understands the inner meaning of things*". *Université François Rabelais* is proud to award the title of *Doctor Honoris Causa* to Professor James SERRIN.

Laurent Véron

All my thanks to those who helped me improve my speech with their relevant suggestions and profound knowledge of Shakespeare's language:

Pierre Damphousse, thank you Pierre

Haïm Brezis, thank you Haïm

Ania Chrusciel, thank you Ania.