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PhD thesis proposal (Sep. 2019/Aug. 2022)

The extinction problem for some populations in biology (see below for a detailed description)

Salary and facilities

The salary offered will be around 17 000 € per year (net), with the possibility to do also some teaching. We will also provide the PhD student with a generous amount of travel money (up to 8 000 € per year if justified) and a laptop if necessary. The funding is provided by the ERC starting grant COMBINEPIC.

How to apply

The candidate should have a strong background in at least one of the following subjects: probability theory, enumerative combinatorics, mathematical physics. Applications (including CV and short description of research interests), as well as the names of two people who can submit letters of recommendation, should be sent before March 31st 2019 by e-mail to gerolda@uni-muenster.de and raschel@math.cnrs.fr.

Objectives of the PhD thesis

A two-site affiliation: It is expected that the PhD student spends half of her/his time at both institutes (Münster and Tours).

Context: The central topic of the PhD thesis is the analysis of the behavior of multitype populations that are modeled by random processes in the orthant $\mathbb{N}^d$ of d-tuples of nonnegative integers. Of particular interest are the computation or approximation of extinction probabilities, the expected number of individuals of a given type, etc., see [2, 6] for some typical examples in the context of flower populations.

From a mathematical point of view, the evolution of multitype populations gives rise to the study of (spatially) inhomogenous random walks, which are profoundly different from homogenous walks (see examples on Figure 2). To handle such processes it is typically assumed in the literature that the drift is going asymptotically to zero, or any similar hypothesis moving the model closer to the homogenous case, see [7]. An important class to be studied in this PhD project may be described as nonhomogenous random walks on $\mathbb{N}^d$ with homogenous branching. Such walks are characterized by a jump mechanism that consists of two parts: one that determines with constant probabilities whether a birth or a death occurs (homogenous part), and a second one that picks a component in the current location vector $x = (x_1, \ldots, x_d)$, say, to be raised by one (birth) or reduced by one (death), with a probability that may depend on x and also on whether a birth or death occurs (nonhomogenous part). The second step corresponds to the type selection.

In many species there are only two classes of reproduction, and sexual reproduction can happen only between individuals of different classes. For such populations (equivalent to random walks in the quarter plane), it becomes particularly relevant to compute extinction probabilities.

The above paragraph is concerned with the case of dimension two, but the problem can easily be stated in a more general framework, since many species have more than two classes of reproduction. For such populations, the computation of extinction probabilities is of crucial interest on the one hand, but mathematically very challenging on the other hand. Another quantity of interest is the probability to pass from d classes to $d-1$ classes.

A rich variety of approaches: There is a large number of inhomogenous random walks corresponding to populations with various biological interpretations. In the recent past, there has been a significant number of different approaches to attack the above problems: partial differential equations [2, 6], potential theory [1] (construction of sub/super-harmonic functions), using reasonings appearing in the study of urns, branching processes, discrete orthogonal polynomials [3], etc.

Detailed objectives: The main goal of the PhD thesis is to unify already existing techniques as well as to develop new techniques to answer the above questions.

Links with other topics: During the course of the PhD, the student will be encouraged to develop interactions with other domains, in which the study of inhomogenous random walks is also relevant. This is the case in combinatorics [4], potential theory, diffusions, queueing theory [5].

References

[1] G. Alsmeyer and K. Raschel (2018). The extinction problem for a distylous plant population with sporophytic self-incompatibility. *arXiv:1805.03123* 1–31 (to appear in *J. Math. Biol.*)

[2] S. Billiard and V. C. Tran (2012). A general stochastic model for sporophytic self-incompatibility. *J. Math. Biol.* **64** 163–210

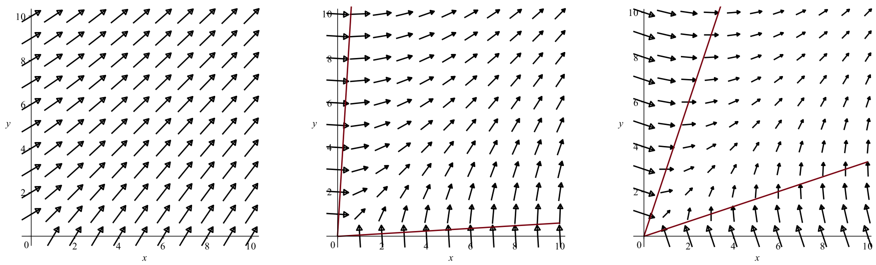

Figure 1: Local evolutions of some population models studied in [6, 1]

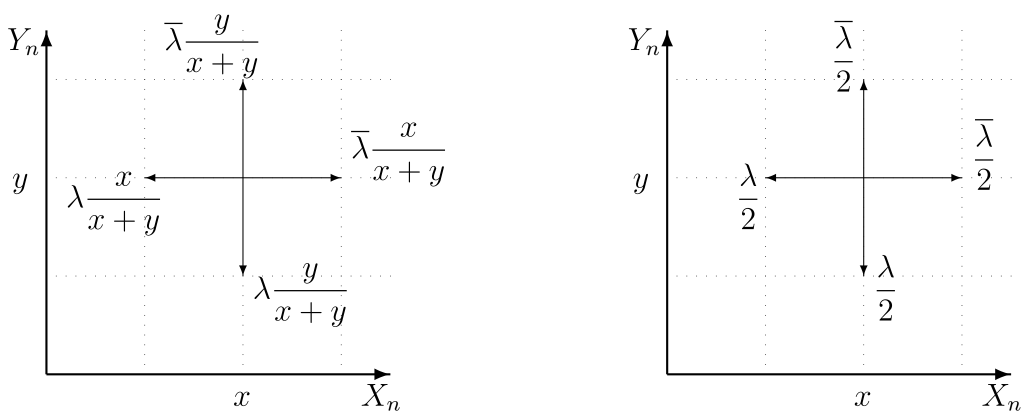

Figure 2: Transition probabilities at (x, y) in a branching model with complete segregation (left panel) and a model with fully symmetric type selection (right), see [6, 1]

[3] N. Champagnat, P. Diaconis and L. Miclo (2012). On Dirichlet eigenvectors for neutral two-dimensional Markov chains. *Electron. J. Probab.* **17** no. 63, 41 pp

[4] P. D'Arco, V. Lacivita and S. Mustapha (2016). Combinatorics meets potential theory. *Electron. J. Combin.* **23** no. 2.28, 17 pp

[5] I. Kurkova and Y. Suhov (2003). Malyshev's theory and JS-queues. Asymptotics of stationary probabilities. *Ann. Appl. Probab.* **13** 1313–1354

[6] P. Lafitte-Godillon, K. Raschel and V. C. Tran (2013). Extinction probabilities for a distylous plant population modeled by an inhomogenous random walk on the positive quadrant. *SIAM J. Appl. Math.* **73** 700–722

[7] M. Menshikov, S. Popov and A. Wade (2017). *Non-homogenous random walks. Lyapunov function methods for near-critical stochastic systems.* Cambridge University Press, Cambridge