

Large Random Networks and Constrained Random Walks

Symposium in honor of the 75th birthday of Guy Fayolle

Dijon / August 27–28, 2018



Foreword

In his Project at INRIA and his recent activity, Guy strongly developed various fields of mathematics and their applications to stochastic networks, using probabilistic, analytic and algebraic techniques. Although I started the adventure about “random walks inside quarter plane” earlier in Moscow, this adventure would never have survived without interference of Guy and Rudolf Iasnogorodski. With them we discussed relations between various fields of mathematics (complex analysis, integral equations, Galois theory etc.) and the role they could play in applied science. I also would like to mention some important results obtained by Guy on the classification of countable Markov chains (ergodicity, recurrence, transience).

Indeed, it could take too much place to describe all achievements of Guy in these fields, but even this is not his most important contribution to science as a whole. I will explain now what I mean by this.

Sometimes it is necessary to look at yourself, at the life and at mathematics from various distances. From very high you see mathematics as a community of people, joined by a special language – mathematical language. And one could think that there should exist a kind of sympathy, of mutual understanding, between the adepts of such language. And that the mathematical community should be overfull with such support and help. But, as we discussed with Guy, mathematics in XXIth century became a much wider field of science if compared to the number of mathematicians (digging local holes). Thus the distances between diggers become so big that there is danger that relations between different fields of mathematics could be lost, and the main goals will be prizes, ratings, impact factors etc. The ugliest thing I ever saw are referee reports like this: “your paper is too low level to be published in our high level journal”. Without Guy and discussions with him our journal MPRF, where we wanted to avoid such unkind reports, would never appear.

That is why I always thought that benevolence (*bienveillance*, *доброжелательность*) and the desire to understand other fields of science, is often more important than very complicated results in narrow fields. I was always attracted by such benevolence to people and to science, and I am very grateful to Guy for introducing me to atmosphere of benevolence, hard work and intoxicating freedom in France, at INRIA.

I regret only that there was no enough time to work with Guy on functional equations techniques in the 3 particle problems of quantum physics. But I hope this could occur and our collaboration with Guy will continue in a forthcoming journal “Structure of Mathematical Physics”.

Finally, I wish Guy many years of creative work.

*Vadim Malyshev
(Lomonosov Moscow State Univ.)*

Symposium Program

Monday, August 27

Morning Session

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| 9:00 | Welcome/registration | |
| 9:30 | Kilian Raschel. <i>"A few opening words"</i> | 7 |
| 9:45 | Ellen Saada. <i>"Ergodicity of some dynamics of DNA sequences"</i> | 7 |
| 10:30 | Roudolf Iasnogorodski. <i>"Restricted three particle quantum walk on \mathbb{Z}_+: explicit solution"</i> | 7 |
| 11:15 | Coffee break | |
| 11:45 | Onno Boxma. <i>"An infinite-server system with Lévy shot noise modulation – Moments and asymptotics"</i> | 8 |
| 12:30 | Lunch | |

Afternoon Session

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|-------|--|---|
| 14:00 | Bernard Derrida. <i>"The many faces of the Fisher–KPP equation"</i> | 8 |
| 14:45 | Ed Coffman. <i>"Network signal cascades with applications ranging from entomology to crowd behavior"</i> | 8 |
| 15:30 | Coffee break | |
| 16:00 | End of the sessions | |

Evening

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| 19:30 | Dinner at <i>la Dame d'Aquitaine</i> | |
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Tuesday, August 28

Morning Session

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| 9:00 | Jean-François Le Gall. <i>"Separating cycles and isoperimetric inequalities in the uniform infinite planar quadrangulation"</i> | 9 |
| 9h45 | Isi Mitrani. <i>"Control and optimization of the SRPT service policy by frequency scaling"</i> | 9 |
| 10:30 | Coffee break | |
| 11:00 | Bernard Bercu. <i>"On the multi-dimensional elephant random walk"</i> | 10 |
| 11:45 | Gabriel Ruget. <i>"Les talents, entre individus et collectifs ? — ou : Du métier d'hébergeur de talents"</i> | 10 |
| 12:30 | Lunch | |

Afternoon Session

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| 14:00 | Alin Bostan. <i>"On the finiteness of groups attached to walks in the quarter plane"</i> | 10 |
| 14:45 | Arnaud de La Fortelle. <i>"Probability: theory and practice"</i> | 11 |
| 15:30 | Guy Fayolle. <i>"Closing remarks"</i> | 11 |
| 15:45 | End of the symposium | |

Monday, August 27

Morning Session

9:30

Kilian Raschel

A few opening words

9:45

Ellen Saada (CNRS & Université Paris V)

Ergodicity of some dynamics of DNA sequences

In this joint work with M. Falconnet and N. Gantert, we define Markov processes on configurations of the integer lattice (with values in some finite alphabet) as the superimposition of two dynamics: an evolution of the sites according to a substitution process on the finite alphabet where the rates possibly depend on a finite set of sites, and a “cut-and-paste” circular permutation mechanism with possibly infinite range. The latter can be interpreted as a “random stirring”. We investigate the ergodicity of these processes. Our study is motivated by the dynamics of DNA sequences; we consider three models from mathematical biology, the JC+CpG model and the T92+CpG model, then the RN+YpR model which generalizes the previous two models.

10:30

Roudolf Iasnogorodski (Saint-Petersbourg)

Restricted three particle quantum walk on \mathbb{Z}_+ : explicit solution

We consider 3 particles on \mathbb{Z} . One of them has infinite mass and stands still at 0. The particles interact only if all of them are at the point 0. We give a full description of essential, point and discrete spectra of the corresponding Hamiltonian.

Note: this is joint work with V. Malyshev and A. Zamyatin

11:45

Onno Boxma (Eindhoven University of Technology)

An infinite-server system with Lévy shot noise modulation – Moments and asymptotics

We consider an infinite-server system with as input process a non-homogeneous Poisson process with rate function $\Lambda(t) = a'X(t)$. Here $\{X(t) : t \geq 0\}$ is a generalized multivariate shot-noise process fed by a Lévy subordinator rather than by just a compound Poisson process.

We study the transient behavior of the model, analyzing the joint distribution of the number of customers in the queueing system jointly with the multivariate shot-noise process. We also provide a recursive procedure that explicitly identifies transient as well as stationary moments and correlations. Various asymptotic results are also derived, including heavy-tail and heavy-traffic asymptotics.

Note: this is joint work with Michel Mandjes and Mayank Saxena

Afternoon Session

14:00

Bernard Derrida (Collège de France)

The many faces of the Fisher–KPP equation

The Fisher–KPP equation describes the growth of a stable region into an unstable medium. It was introduced in 1937 both by the biologist and statistician Fisher and by the mathematicians Kolmogorov, Petrovsky, Piscounov to describe the propagation of a favorable gene in a population. It is one of the classical examples of the problem of velocity selection. It also appears in many other contexts, ranging from the theory of disordered systems and spin glasses to reaction diffusion problems, branching Brownian motion and models of evolution with selection. This talk will try to review the main classical results on this equation as well as some recent progress.

14:45

Ed Coffman (Columbia University)

Network signal cascades with applications ranging from entomology to crowd behavior

I'd like only to set the problems in the most general way – no results beyond handwaving. I'd wind up with amusing anecdotes about Guy and me.

Tuesday, August 28

Morning Session

9:00

Jean-François Le Gall (Université Paris-Sud)

Separating cycles and isoperimetric inequalities in the uniform infinite planar quadrangulation

The uniform infinite planar quadrangulation or UIPQ is a random planar lattice whose faces are quadrangles arranged in a random manner. It has been known for some time that the volume of the ball of radius r centered at the origin of the UIPQ grows like r^4 , whereas the perimeter grows like r^2 . We investigate the minimal length of a cycle separating the ball of radius r from infinity, and we establish a precise form of a conjecture of Krikun stating that this minimal length grows linearly in r . As an application, we prove isoperimetric inequalities showing that for any connected union of faces containing the origin, the size of the boundary is at least the volume to the power $1/4$, up to logarithmic corrections. This talk is based on a joint work with Thomas Lehericy.

9h45

Isi Mitrani (Newcastle University)

Control and optimization of the SRPT service policy by frequency scaling

A system where the speed of a processor depends on the current number of jobs is studied by analysing appropriate queueing models. Jobs consist of random numbers of tasks and priority is given to the job with fewest remaining tasks. The number of possible frequency levels of the processor determines the dimensionality of the queueing process. Exact solutions are obtained in the cases of 2 and 3 frequency levels, and accurate approximations are developed that may be generalized to higher dimensions.

Note: this is joint work with Andrea Marin, Maryam Elahi and Carey Williamson

11:00

Bernard Bercu (Institut de Mathématiques de Bordeaux)

On the multi-dimensional elephant random walk

Random walks with long-memory arose naturally in mathematical physics and econometrics. One of them is the so-called elephant random walk. It is a discrete-time random walk on integers, which has a complete memory of its whole history. The asymptotic behavior of this random walk mainly depends on a memory parameter p lying between zero and one. This behavior is totally different in the diffusive regime $0 \leq p < p_d$, the critical regime $p = p_d$, and the super-diffusive regime $p_d < p \leq 1$, where the value p_d can be explicitly calculated. In this talk, we establish the almost sure convergence and the asymptotic normality of the multi-dimensional elephant random walk. All our analysis relies on asymptotic results for martingales.

11:45

Gabriel Ruget (Ancien directeur de l'ENS)

Les talents, entre individus et collectifs ? — ou : Du métier d'hébergeur de talents

Entre quelques expériences personnelles dans les milieux académiques et industriels, et emprunts substantiels au cours 2017/2018 de Pierre-Michel Menger au Collège de France, je parcourrai rapidement la distance entre la conception du talent chez Aristote ou Durkheim et les talents à gratification « disproportionnelle » des premiers de cordées mondialisées — sans oublier la vision marxienne du talent et son éventuel domaine de pertinence aujourd'hui.

Afternoon Session

14:00

Alin Bostan (Inria Saclay)

On the finiteness of groups attached to walks in the quarter plane

We revisit several aspects of recent results on walks confined to the quarter plane, notably regarding the finiteness of the associated groups, with an emphasis on Guy's contributions to the topic.

14:45

Arnaud de La Fortelle (MINES ParisTech)

Probability: theory and practice

Guy Fayolle did not only focus on large stochastic networks and constrained random walks during his career. This talk intends at giving credits to Guy to quite a few application of the probabilistic concepts he developed as well as he taught. From my own experience, together with Guy — still my PhD supervisor in my heart — this will focus on transportation problems, which is a wonderful playground for probabilities, and not only because transport is chaotic, with a large network and times of forced walks especially in a striking France. This talk will focus on a few concrete recent examples where new tools and concepts really need new developments.

15:30

Guy Fayolle

Closing remarks