The stationary tail index of contractive iterated function systems

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Abstract

Let $(X_n)_{n \geq 0}$ be a contractive iterated function system (IFS) on a complete separable metric space $(\mathbb{X}, d)$ with unbounded metric d, i.e.

$$X_n = \Psi_n \circ ... \circ \Psi_1(X_0)$$

for $n \geq 1$, where $\Psi_1, \Psi_2, ...$ are iid random Lipschitz functions on $\mathbb{X}$ with Lipschitz constants $L(\Psi_1), L(\Psi_2), ...$ Let π denote the unique stationary distribution of $(X_n)_{n \geq 0}$ and $x_0 \in \mathbb{X}$ an arbitrary reference point. Assuming $\mathbb{P}_\pi(d(x_0, X_0) > r) > 0$ for all $r > 0$, we will provide bounds for the lower and upper tail index ϑ_* and ϑ^* of $d(x_0, X_0)$ in equilibrium (under $\mathbb{P}_\pi$), defined by

$$\vartheta_* := -\limsup_{x \to \infty} \frac{\log \mathbb{P}(X > x)}{\log x} \quad \text{and} \quad \vartheta^* := -\liminf_{x \to \infty} \frac{\log \mathbb{P}(X > x)}{\log x}.$$

This will be done by providing lower and upper bounds for $d(x_0, X_n)$ under $\mathbb{P}_\pi$ in terms of rather simple IFS on $\mathbb{R}_\geq$ and the use of Goldie's implicit renewal theorem [3]. Special attention is paid to the particularly relevant case when $\mathbb{X} = \mathbb{R}$. The method is illustrated by some examples including the well-known AR(1) model with ARCH(1) errors which has been studied earlier in some detail by Borkovec and Klüppelberg [2].

References

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