Return Probabilities for the Reflected Random Walk on $\mathbb{N}_0$

ERRATUM

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In the above mentionned paper, we fixed a constant $\mathbf{K} > 1$ and consider the set $\mathcal{K}(\mathbf{K})$ of functions $K : \mathbb{Z} \to \mathbb{R}^+$satisfying the following conditions

$$\forall x \in \mathbb{N}_0 \quad K(x) \geq 1, \quad \mathcal{R}K(x) \leq 1 \quad \text{and} \quad K(x) \sim \mathbf{K}^x. \tag{1}$$

Unfortunately, the two first conditions readily imply $K = 1$ since the operator $\mathcal{R}$ is markovian , so that the 3 above conditions cannot be satisfied simultaneously.

In fact, we will simply consider the function $K : s \mapsto \mathbf{K}^x$. The only one reason of the condition $\mathcal{R}K(x) \leq 1$ appeared in the proof of Fact 4.4.1, where the peripherical spectrum of the operators $\mathcal{R}_s$ for $|s| = 1$ and $s \neq 1$ is controlled. With this new choice of function K, one gets the

Fact 4.4.1- *For $|s| = 1$ and $s \neq 1$ one gets $\|\mathcal{R}_s\|_K < 1$; in particular, the spectral radius of $\mathcal{R}_s$ on $(\mathbb{C}_0^{\mathbb{N}}, \|\cdot\|_K)$ is < 1.*

Proof. We could adapt the proof proposed in the paper and show that $\|\mathcal{R}_s^{2n}\|_K \leq C\rho_s^n$ for some $\rho_s < 1$ when $s \neq 1$. We propose here another simpler argument.

Recall that $\mathcal{R}_s$ acts from $(\mathbb{C}^N, |\cdot|_K)$ into $(\mathbb{C}^N, |\cdot|_\infty)$ and that the identity map is compact from $(\mathbb{C}^N, |\cdot|_\infty)$ into $(\mathbb{C}^N, |\cdot|_K)$. Consequently, the operator $\mathcal{R}_K$ is compact on $(\mathbb{C}^N, |\cdot|_K)$ with spectral radius ≤ 1 since it has bounded powers.

Let us fix $s \in \mathbb{C} \setminus \{1\}$ with modulus 1 and assume that $\mathcal{R}_s$ has spectral radius 1 on $(\mathbb{C}^N, |\cdot|_K)$; since it is compact, there exists a sequence $\mathbf{a} = (a_x)_{x\in\mathbb{Z}} \neq 0$ and $\theta \in \mathbb{R}$ such that $\mathcal{R}_s\mathbf{a} = e^{i\theta}\mathbf{a}$, i.e.

$$\forall x \in \mathbb{Z} \qquad \sum_{y\in\mathbb{Z}} \mathcal{R}_s(x,y)a_y = e^{i\theta}a_x. \tag{2}$$

It follows that $|a_y| = |a_0| \neq 0$ for any $y \in \mathbb{Z}$ since $\sum_{y\in\mathbb{Z}} |\mathcal{R}_s(x,y)| \leq \sum_{y\in\mathbb{Z}} \mathcal{R}(x,y) = 1$; without loss of generality, we may assume $|a_y| = 1$, i.e. $a_y = e^{i\alpha_y}$ for some $\alpha_y \in \mathbb{R}$. The equality (2) may be thus rewritten

$$\forall x \in \mathbb{Z} \qquad \sum_{y\in\mathbb{Z}} \mathcal{R}_s(x,y)e^{i\alpha_y} = e^{i\theta}e^{i\alpha_x}.$$

By convexity, using again the inequality $\sum_{y\in\mathbb{Z}} |\mathcal{R}_s(x,y)| \leq \sum_{y\in\mathbb{Z}} \mathcal{R}(x,y) = 1$, one readily gets $e^{i\alpha_y} = e^{i\theta}e^{i\alpha_x}$ for any $x, y \in \mathbb{Z}$; consequently $e^{i\theta} = 1$, the sequence $\mathbf{a}$ is constant and $R_s(x,y) = \mathcal{R}(x,y)$ for any $x, y \in \mathbb{Z}$, which implies in particuler $s = 1$. Contradiction.□

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