From dying massive stars to nascent compact objects: proto-neutron stars

Loïc Villain
Departament d'Astronomia i Astrofisica (DAA)
Universitat de Valencia
Valencia, Spain
and
Laboratoire de l'Univers et de ses THéories (LUTH)
Observatoire de Paris-Meudon, France.

Based on a collaboration with
Jose Antonio Pons, Pablo Cerdá-Durán & Eric Gourgoulhon

Seminar at LMPT, Tours

Plan

- Brief introduction to proto-neutron stars (PNS):
  - rough description : **birth and main features**
  - principles of the **time evolution**

- Modelisation:
  - hypothetical almost full modelisation
  - retained simplified modelisation

1

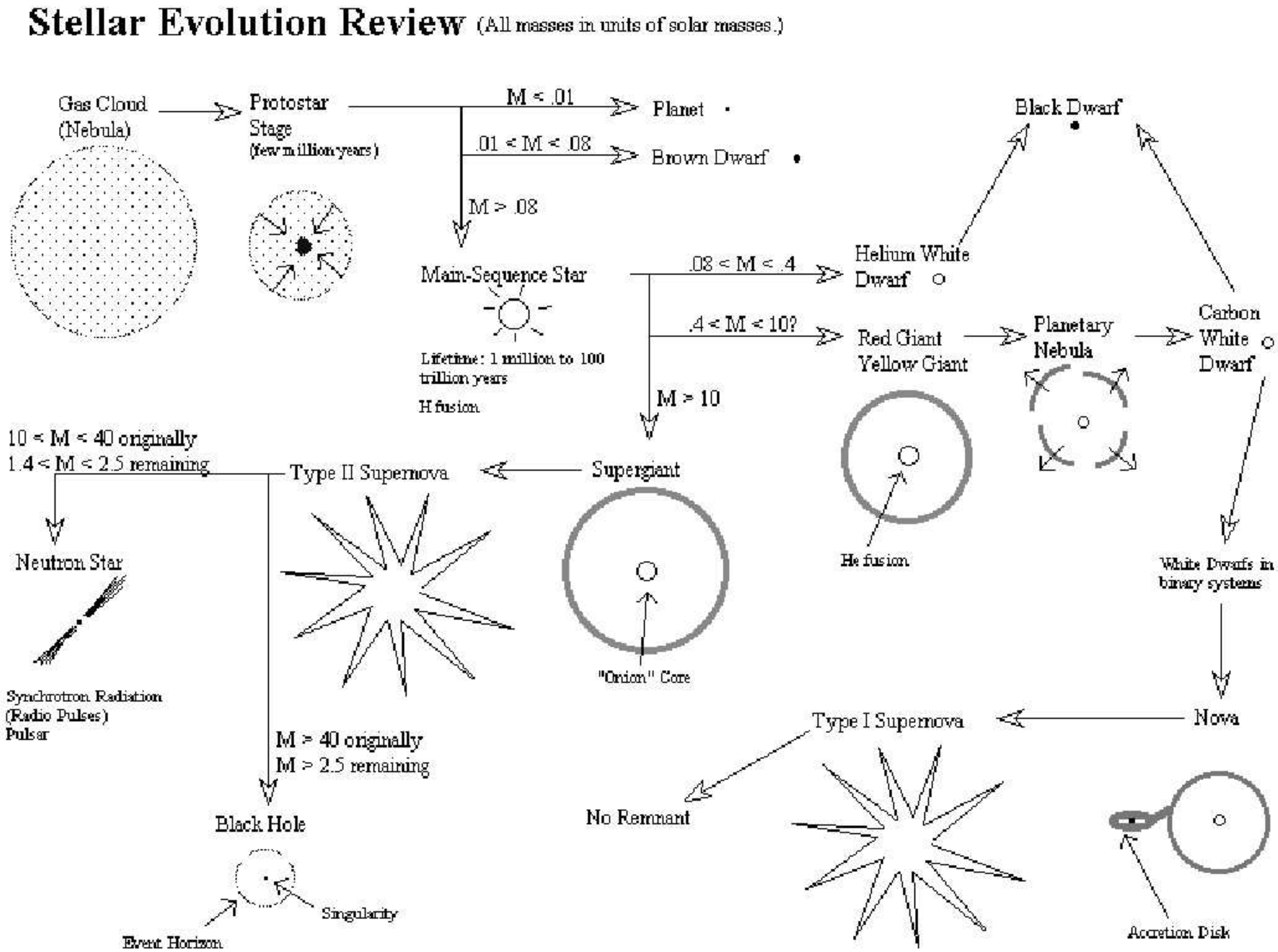
Brief introduction to proto-neutron stars
A: general overview

Definition and origin(s) of proto-neutron Star (PNS)

- **Definition:**
  - **PNS** $\sim$ hot ($T \sim 50\,\mathrm{MeV} \sim 6.10^{11}$ K) **neutron stars (NS)** that **may** become cold ($\sim$ degenerate since T ~ 4 MeV) **NS**
  - typical **timescale of the evolution:** 20 seconds
  - only "may": **black holes or strange stars** possible too (not here);

- **Birth:**
  - **standard scenario:** gravitational collapse of the iron core of a massive star $(M \geq 10 - 15\,M_\odot)$ $\rightarrow$ Supernova $+$ PNS
  - **alternative scenarios:** non-explosive collapse of white dwarves, merging of binary neutron stars.

Standard scenario for birth (Source J. Lewis)



Why studying proto-neutron stars?

- link between old massive stars and compact objects $\longrightarrow$ initial state of compact objects? (crucial for the emission of gravitational waves)
- emission of neutrinos during the cooling ($\sim 99\,\%$ of the neutrinos emitted by a SN)
- source of gravitational waves? $\longrightarrow$ way of putting constraints on equation of state of nuclear matter.

Main parameters

- after collapse: central remnant = hot soup of neutrons, protons, electrons and **neutrinos** (2 main regions);

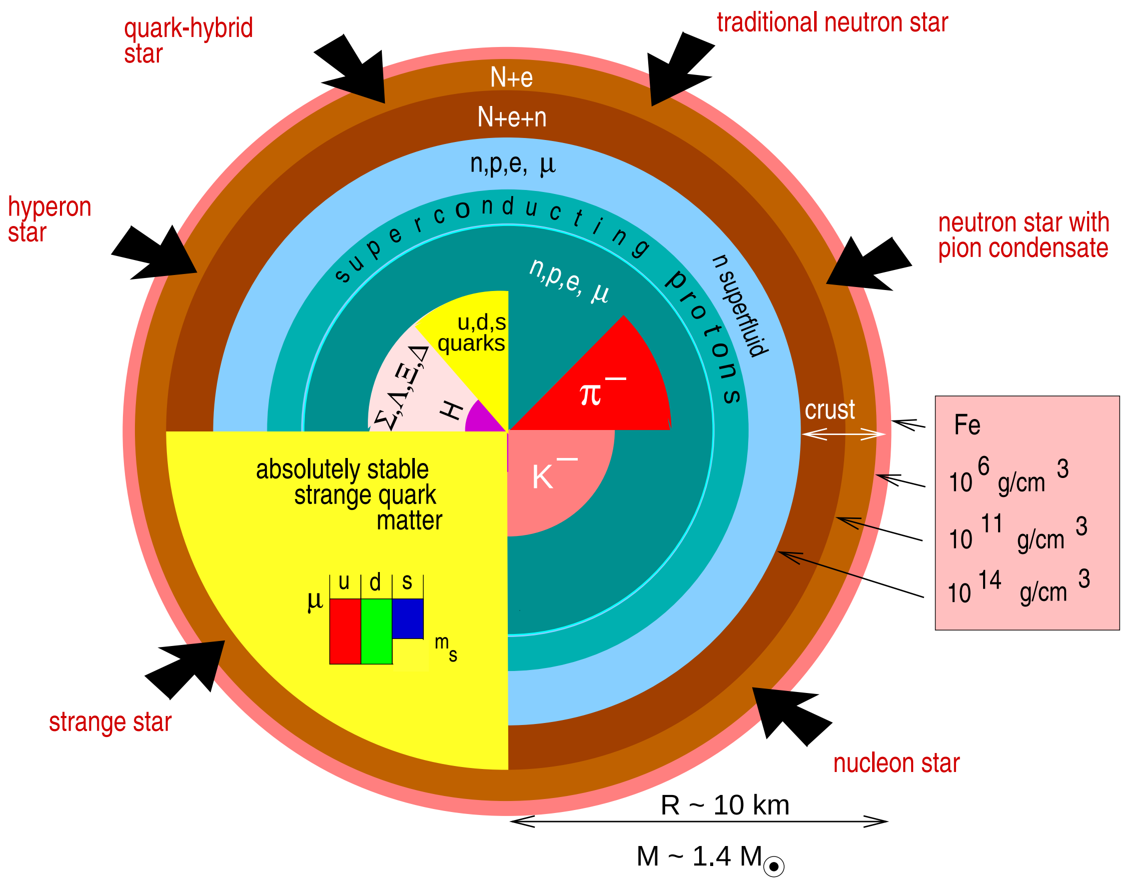
- $T \sim 6.10^{11}$ K: $npe\nu$ matter **opaque** for neutrinos but cooling down;

- mass from 1.2 to $2\ M_\odot$ ($\sim$ mass of the core) ;

- radius from 10^2 km to 10 km: evolves during the "20 seconds" of life

  $\rightarrow$ compacity $(M/R) = 0.1$ after 10 seconds
  (Sun: $\sim 10^{-6}$; Schw. BH: $R = 2\,M$)

Pictures of cold compact stars (Source F. Weber)



Possible consequence: **phase transitions** during contraction/cooling

1

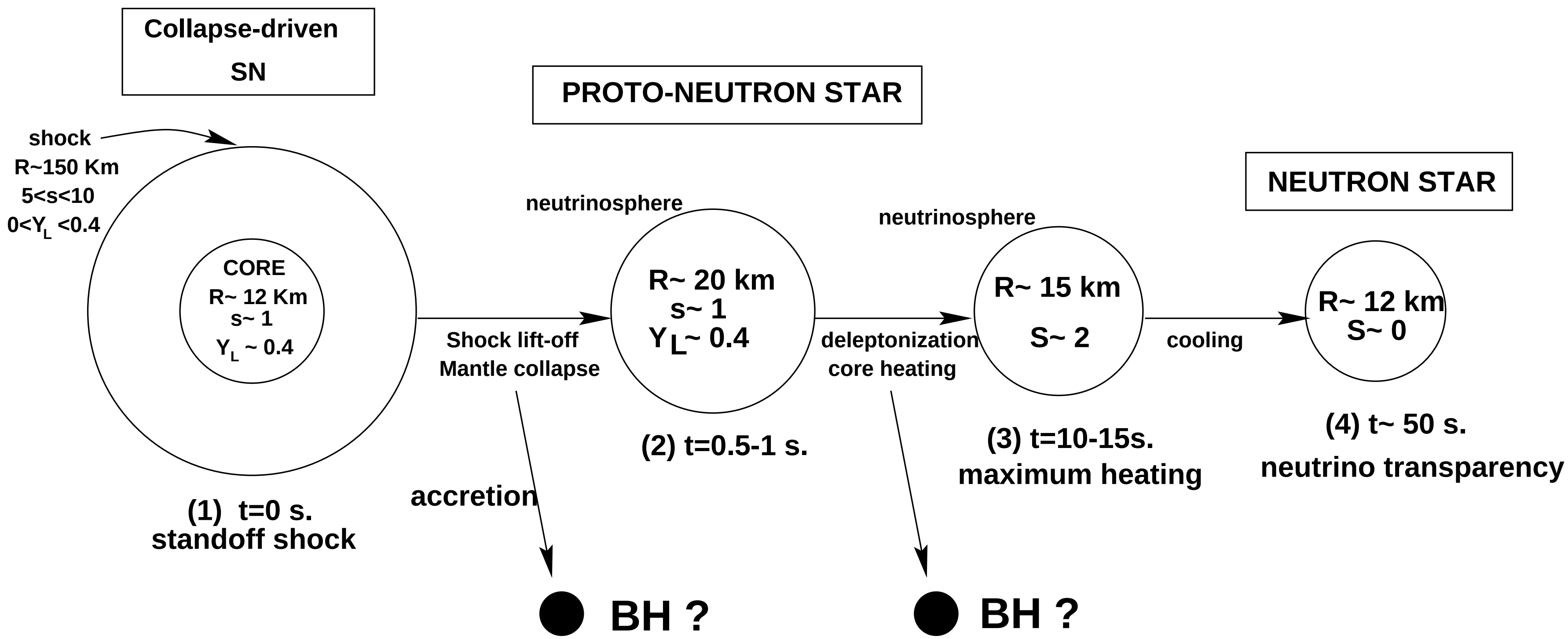
Brief introduction to proto-neutron stars
B: evolution

Standard scenario for evolution (1)

Main stages [see review by Prakash *et al.* (2001)]:

- after collapse: central remnant = hot soup of neutrons, protons, electrons and **neutrinos** (2 main regions);
- mantle contraction: fast cooling of **outer layers**, accretion ($t < 0.5$ s);
- deleptonization: neutrinos diffusing out $\longrightarrow$ heating of the **inner part** ($t < 10 - 15$ s);
- cooling: diffusion of quasi-thermal neutrinos ($t < 20 - 25$ s);
- birth of a cold compact object.

Standard scenario for evolution (2)



references (non-rotating PNS): Burrows & Lattimer (1986); Keil & Janka (1995); Pons *et al.* (1999).

Questions

- effect of the rotation? rotation profile?

- angular momentum: conserved during core collapse? role of magnetic field?

- how does rotation influence the propagation of neutrinos, heat and angular momentum?

  $\rightarrow$ Romero *et al.* (1992); Goussard *et al.* (1997,98); Sumiyoshi *et al.* (1999); Strobel *et al.* (1999); Yuan & Heyl (2003)

  but all works with simplified thermodynamics (constant profiles or even $T = 0$) and without time evolution (few stationnary states calculated only).

2

Toward an understanding of the evolution of proto-neutron stars

A: general overview

Indications for a recipy of an almost complete modelisation

• relativistic modeling of rotating contracting star (>1 dimension due to rotation) and of spacetime

• self-consistent description of a non-perfect fluid at supra-nuclear densities (plasma of n, p, e^-, ν + possibly exotic particles and phase transitions)

• non-zero temperatures (matter opaque for neutrinos)

• transport of heat and neutrinos while the rotating star contracts and cools down (Boltzmann + hydrodynamics)

• non-trivial initial state of rotation (differential rotation)

• changing surrounding (accretion during the first seconds)

$\longrightarrow$ need to start with simplified models.

First simplified modelisation [Villain *et al.* (2004)]

• gravitational relaxation time $\lesssim$ dynamical time $\ll$ diffusion time
$\longrightarrow$ mimicking time evolution with sequence of quasi-equilibrium states
$\longrightarrow$ for each of them, stationary and axisymmetric spacetime;

• non-trivial description of thermodynamics: transport of neutrinos and heat in spherical self-consistent calculations [mean free paths of ν hugely dependent on microphysics: Pons *et al.* (1999)]
$\longrightarrow$ time dependent equation of state with non-constant profiles;

• profile of rotation in agreement with core collapse simulations;

• **constraints on the sequences**: conservation of angular momentum(?) and gravitational mass $\equiv$ energy with losses due to ν luminosity.

2

Toward an understanding of the evolution of proto-neutron stars

B: some technical details: macroscospic point of view

Retained relativistic description

• spacetime is stationary and axisymmetric $\rightarrow$ two Killing vectors (metric depends only on r and θ)

• no meridional convective currents [quite strong assumption Keil *et al.* (1996), Miralles *et al.* (2002)] **but** simplifies the metric (Carter 1969)

$$ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu = -\left(N^2 - N_\phi\, N^\phi\right) dt^2 - 2\, N_\phi\, dt\, d\phi$$
$$+ \tfrac{A^4}{B^2} \left(dr^2 + r^2\, d\theta^2 + B^4\, r^2\, \sin^2\theta\, d\phi^2\right)$$

• metric with notations of $3+1$ formalism : splitting of spacetime in spacelike slices (maximal slicing-quasi-isotropic coordinates, Bonazzola *et al.* 1993) : N lapse, N^ϕ third component of shift 3-vector, with $N_\phi = g_{\phi i}\, N^i = A^4\, B^2\, N^\phi r^2\, \sin^2\theta$.

Solving of Einstein equations

- only elliptic equations for the metric $\longrightarrow$ spectral code with compactification of spacetime (exact boundary conditions)

- spectral $\equiv$ solving of partial differential equations in reciprocal space after Fourier, Legendre and Chebyshev transformations (for spherical symmetry):

$$(r, \theta, \phi) \rightarrow (n, l, m)$$

  $\longrightarrow$ linear algebra

- code LORENE in the public domain: http://www.lorene.obspm.fr

Dealing with the Energy-Momentum tensor (1)

• axisymmetry, stationarity and $3+1$ foliation :
$\nabla_\mu T^{\mu\nu} = 0$ for perfect fluid $\leftrightarrow$ conditions for a first integral

$$\frac{\partial_i P}{e + P} + \partial_i \ln\left[\frac{N}{\Gamma}\right] = -F\,\partial_i\,\Omega\,,$$

where:
* P pressure and e total energy density (in the comoving frame)
* N lapse function (gives gravitational potential), Γ Lorentz factor between the coordinate and comoving observers, Ω angular velocity, and $F = u_\phi\,u^t$ with u 4-velocity of the fluid
* the index i runs from 1 to 2 (all quantities depend only on r and θ).

Dealing with the Energy-Momentum tensor (2)

• **relativistic equation**:

$$\frac{\partial_i P}{e + P} + \partial_i \ln\left[\frac{N}{\Gamma}\right] = -F \partial_i \Omega ,$$

• **Newtonian limit**:

* $\ln[N] \longrightarrow U$;
* $e + P \longrightarrow n$;
* $\ln[\Gamma] = \ln[(1 - v^2)^{-1/2}] \longrightarrow \frac{1}{2} v^2$;
* $F \sim$ specific angular momentum $\longrightarrow \Omega\, \mathrm{r}^2 \sin^2[\theta]$;

$$\longrightarrow \qquad \frac{\vec{\nabla} P}{n} + \vec{\nabla} U - \frac{1}{2} \vec{\nabla} v^2 = -\Omega\, r^2 \sin^2[\theta]\, \vec{\nabla}\Omega ,$$

Dealing with the Energy-Momentum tensor (3)

• right-hand side : rigid rotation ($\partial_i \Omega = 0$) or differential rotation with locally $F = F[\Omega]$ (Newtonian case $\Omega = \Omega[r \sin[\theta]]$);

• left-hand side : need to give an equation of state (EOS) with either
* isentropic profile (can be shown using principles of thermodynamics)
* relativistic isothermal profile (same remark)
* effective polytrop with $e = e[P]$

• $e = e[P] \rightarrow H + \ln\left[\frac{N}{\Gamma}\right] + \int F[\Omega]\, d\Omega = \mathrm{const.}$
relativistic pseudo-enthalpy $H = \int \frac{dP}{e[P] + P}$ with boundary conditions

• note : $T = 0 \rightarrow H$ relativistic enthalpy $H = \ln\left[\frac{e + P}{n_b}\right]$

2

Toward an understanding of the evolution of proto-neutron stars
C: some technical details: microscospic point of view

Retained microphysical description (1)

• Walecka-type relativistic field theoretical model: interactions mediated by σ, ω, and ρ **mesons** + scalar non-linear self interactions $U(\sigma)$
• Lagrangian density: **nucleons**, **leptons** + mesons (no exotic particles)

$$
\begin{aligned}
L_H &= \sum_B \overline{B}(-i\gamma^\mu\partial_\mu - g_{\omega B}\gamma^\mu\omega_\mu - g_{\rho B}\gamma^\mu \mathbf{b}_\mu \cdot \mathbf{t} - M_B^*)B \\
&- \frac{1}{4}W_{\mu\nu}W^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - \frac{1}{4}\mathbf{B}_{\mu\nu}\mathbf{B}^{\mu\nu} + \frac{1}{2}m_\rho^2 b_\mu b^\mu \\
&+ \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2}m_\sigma^2\sigma^2 - U(\sigma) \\
L_\ell &= \sum_l \bar{l}(-i\gamma^\mu\partial_\mu - m_l)l \ \rightarrow \ L = L_H + L_\ell + ...
\end{aligned}
$$

Retained microphysical description (2)

- coupling constants = major uncertainty
- **nucleon-meson** coupling constants adjusted to reproduce properties of equilibrium nuclear matter at $T = 0$.

* the saturation density, n_0
* the binding energy/particle, E_A
* the compression modulus, K
* the symmetry energy coefficient, S_{sym}
* the Dirac effective mass at saturation, M^*

K_0	a_{sym}	$\frac{M^*}{M}$	$\frac{g_\sigma}{m_\sigma}$	$\frac{g_\omega}{m_\omega}$	$\frac{g_\rho}{m_\rho}$	b	c
300	32.5	0.70	3.434	2.674	2.100	0.002950	-0.00107
240	32.5	0.78	3.151	2.195	2.189	0.008659	-0.002421
240	27.5	0.78	3.151	2.195	1.862	0.008659	-0.002421

Coupling constants for the Glendenning & Moszkowski (1991) models.

Retained microphysical description (3)

• partition function + charge neutrality + chemical equilibrium $\rightarrow$ thermodynamical quantities and general EOS $P = P(n_B, s, Y_e, Y_\nu)$

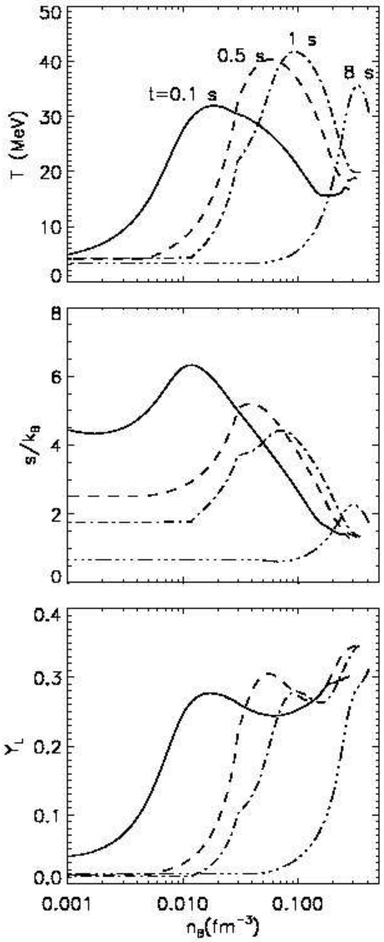
• ν distribution function supposed to be nearly Fermi-Dirac and isotropic $\rightarrow$ dynamic diffusion of ν (and e^- number) and energy:

$$\frac{dY_\nu}{dt} + \frac{\partial(e^\phi 4\pi r^2 F_n)}{\partial a} = \frac{S_n e^\phi}{n_b}$$

$$\frac{dE}{dt} + P\frac{d\left(\dfrac{1}{n_B}\right)}{dt} + e^{-\phi}\frac{\partial(e^{2\phi}4\pi r^2 F_e)}{\partial a} = 0$$

(fluxes obtained from generalized Fick Law consistent with previous EOS)

Time dependent Equation of state



Extract from the final time dependent barotropic EOS: temporal variation of the temperature (top), entropy (middle) and lepton fraction (bottom) as functions of the baryon density, for evolutionary times corresponding to $t = 0.1, 0.5, 1$ and 8 seconds after formation of the PNS. Results are for a $M_B = 1.6\, M_\odot$ star in spherical symmetry. Note: BPS or LS EOS used at low densities.

2

Toward an understanding of the evolution of proto-neutron stars
D: initial state and profile of rotation

Known results

• Pons *et al.* (non-rotating PNS):
* initial profile of T not important;
* baryonic mass important $\rightarrow M_B = 1.6\, M_\odot$
($\rightarrow M_g = 1.4\, M_\odot$ for cold NS)

• **even with** iron core rigidly rotating [Heger *et al.* (2000)] $\rightarrow$ differential rotation after collapse [see also Müller *et al.* (2003)]

calculations done using the code for core collapses from Dimmelmeier *et al.* (2002) $\rightarrow$ "constraints" on the profile of rotation

• **yet** many instabilities to rigidify motion [Miralles *et al.* (2000,2002)]

Law for rotation

- "usual" law for $F[\Omega]$ ($\sim$ specific angular momentum):

$$F[\Omega] = R_0^2\,(\Omega_c - \Omega)\,,$$

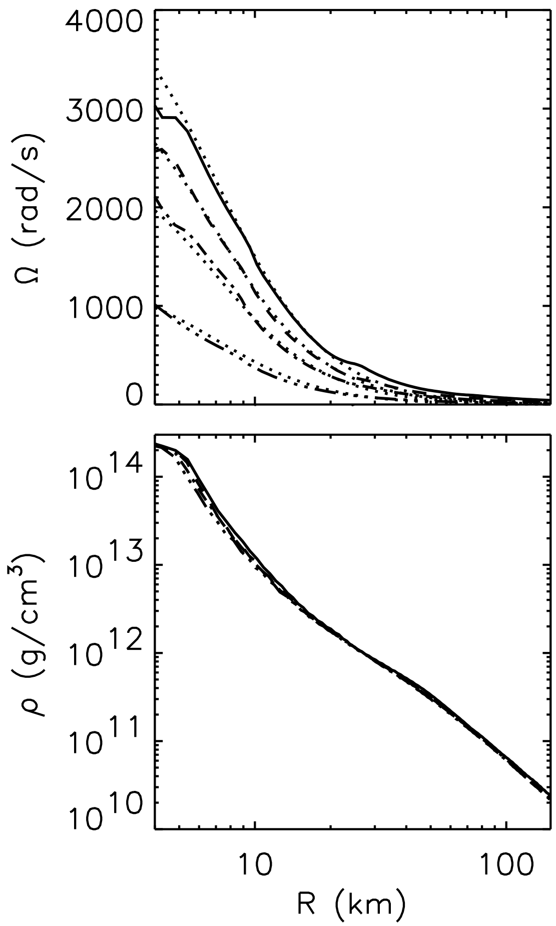
proposed by Komatsu *et al.* (1989)

- simple, realistic (see core collapse simulations) with Newtonian limit :

$$\Omega = \frac{R_0^2\,\Omega_c}{R_0^2 + r^2\,\sin^2\theta}$$

R_0 length describing the degree of "differentiallity" ($R_0 = \infty$: rigid rotation; $r = R_0$: $\Omega = \Omega_c/2$), Ω_c is the "axial" angular velocity.

Calibration of the law for rotation



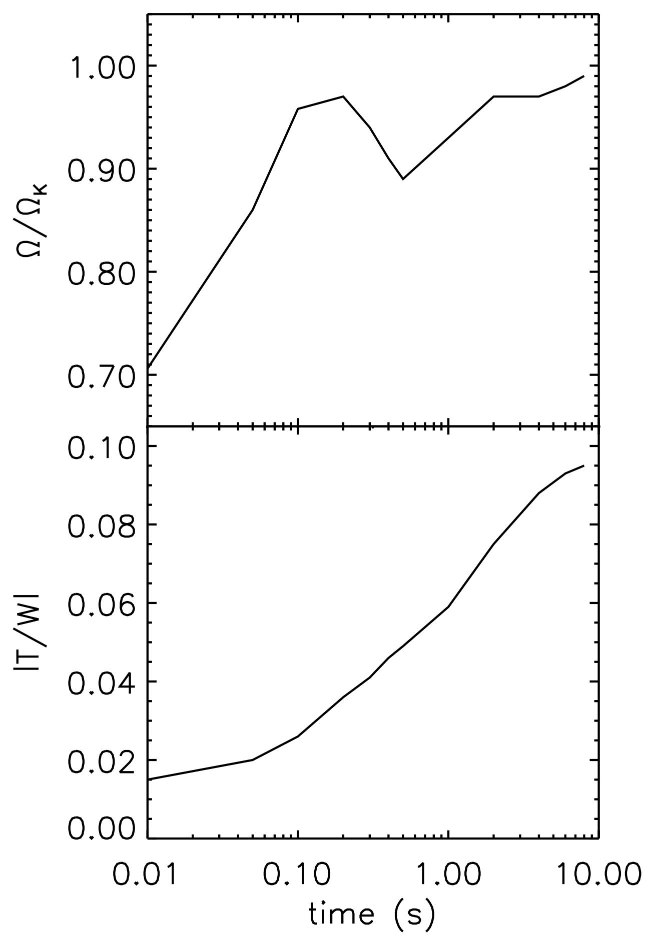
Equatorial profiles of angular velocity (top), and density (bottom) of PNSs from axisymmetric simulations of stellar core collapse (DFM). Models with different amounts of angular momentum of the iron core, namely, $|T/W| = 0.9\%$ (solid), 0.5% (dashes), 0.25% (dash-dot), and 0.05% (dash-3 dots). The dotted lines on the upper panel are fits to the simple law with $R_0^2 = 50$ km^2.

2

Toward an understanding of the evolution of proto-neutron stars

D: influence of the rotation and trying-to-be-realistic thermodynamics

Maximal value of angular momentum (1)



Temporal evolution of the angular velocity Ω/Ω_K and the rotation parameter $|T/W|$ for a sequence of rigidly rotating PNSs with constant angular momentum $(1.5\,GM_\odot^2/c)$ and a fixed baryonic mass of $M_\mathrm{B} = 1.6\,M_\odot$.

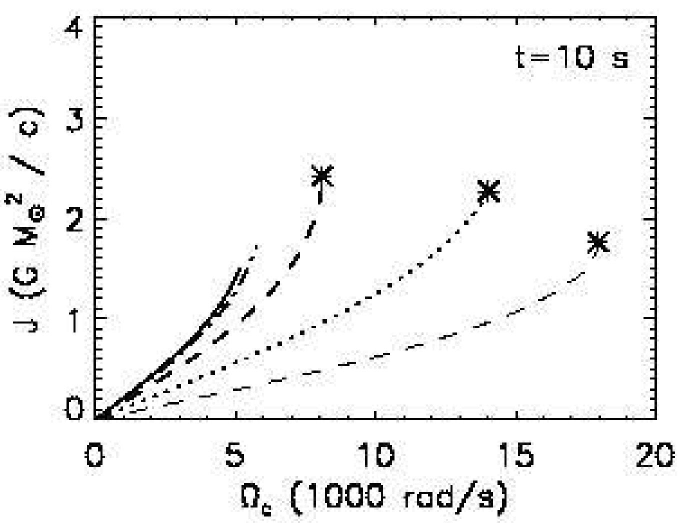
Note 1: For $t < 1$ s: accretion, external convection...

Note 2: $|T/W| = 0.14$ Newtonian criterion for gravitational waves driven instability.

Note 3: $J_\odot \sim 0.2\,GM_\odot^2/c$.

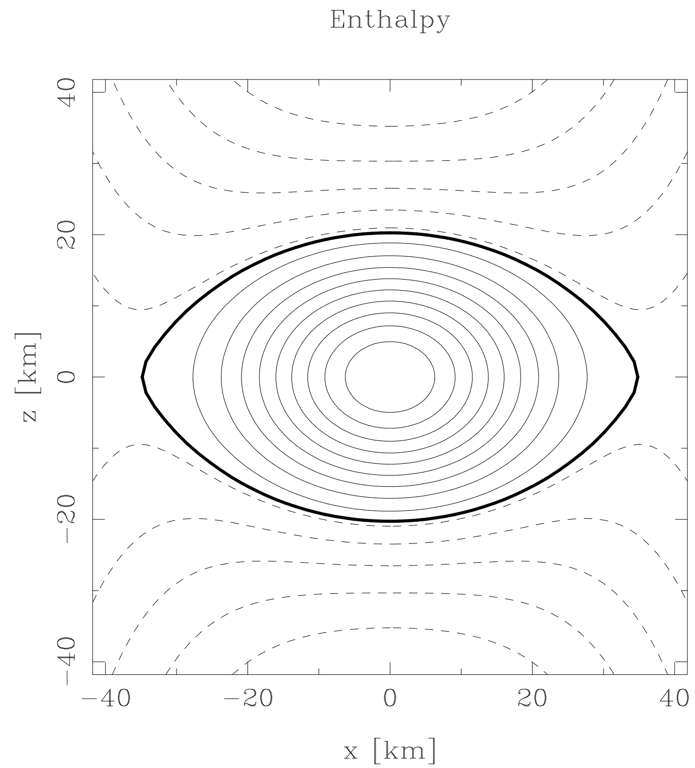
Note 4: recent stellar evolution [Heger *et al.* (2003)] $\rightarrow J = (0.35 - 10)$ (depends on magnetic braking).

Maximal value of angular momentum (2)



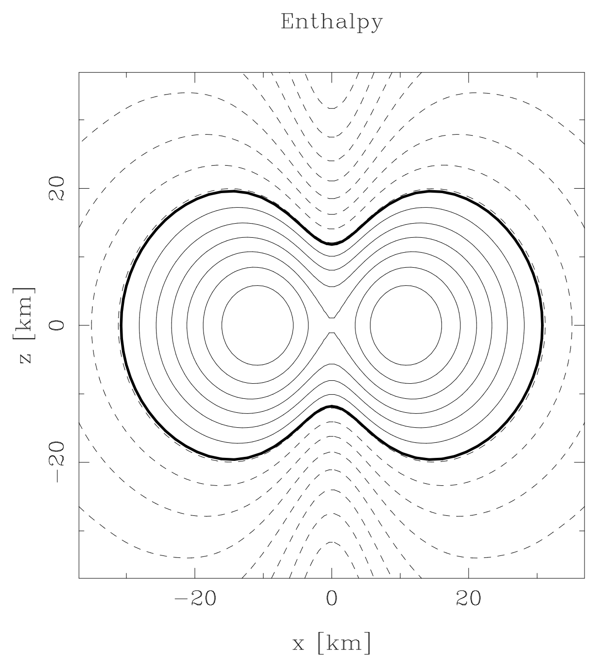
Total angular momentum as a function of the central angular velocity for PNSs with fixed baryonic mass of 1.6 $M_\odot$ at evolutionary time of 10 s for different values of R_0: $R_0 = \infty$ (solid), 50 km (dash-dot), 20 km (thick dashed), $R_{eq}/2$ (thin dashed), 10 km (dots). Differential rotation allows more angular momentum and $|T/W|$ **but...**

Maximal value of angular momentum (3)



“Usual” star at Kepler angular velocity.

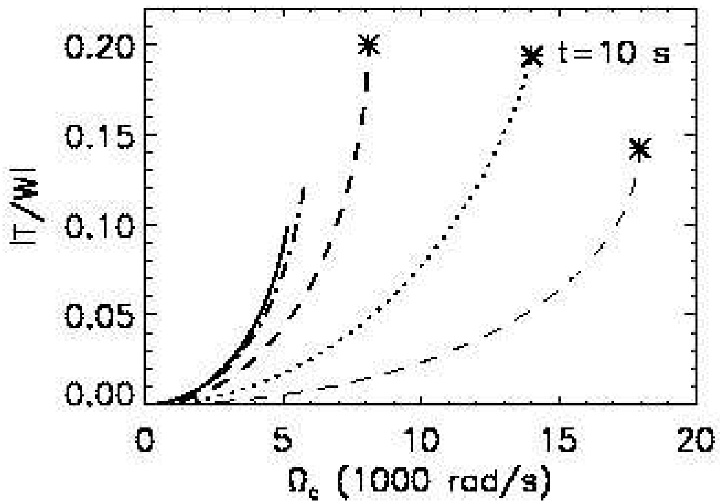
Maximal value of angular momentum (4)



Trying-to-be-toroidal star before Kepler angular velocity.

Consequence: **maximal density off axis** $\longrightarrow$ rotation profile? stability?

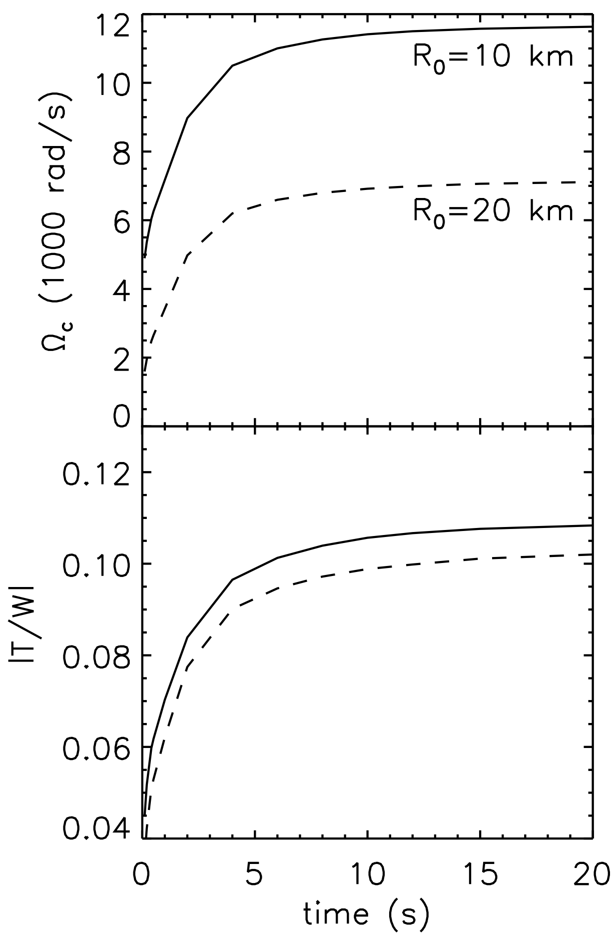
Maximal value of $|T/W|$



Rotation parameter, $|T/W|$, as a function of the central angular velocity for PNS with fixed baryonic mass of 1.6 $M_{\odot}$ at evolutionary time of 10 s for different values of R_0: $R_0 = \infty$ (solid), 50 km (dash-dot), 20 km (thick dashed), $R_{eq}/2$ (thin dashed), 10 km (dots).

Note: with time the criterion for instability decreases due to relativistic effects.

Time evolution of differentially rotating PNS

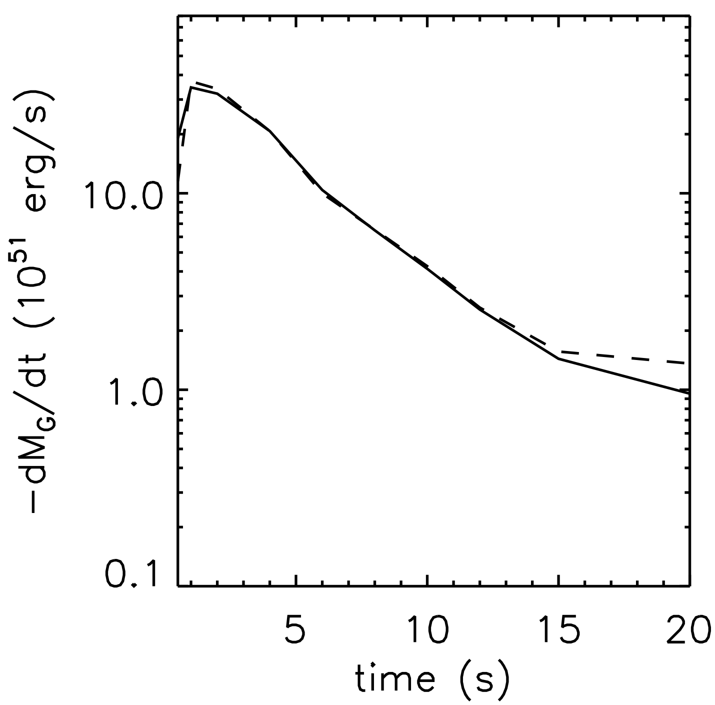


Temporal evolution of the central angular velocity Ω_c and the rotation parameter $|T/W|$ for two sequences of differentially rotating PNSs with $R_0 = 10$ km (solid line) and $R_0 = 20$ km (dashed line). Total angular momentum $1.5\, GM_\odot^2/c$ and baryonic mass $M_\mathrm{B} = 1.6\, M_\odot$.

Note 1: for $R_0 = 10$ km, star always toroidal; for $R_0 = 20$ km, change of convexity;

Note 2: for same value of J, Kepler frequency reached with rigid rotation.

Time evolution of neutrino luminosity



Estimated neutrino luminosity ($-dM_G/dt$). Order of magnitude and exponential decay similar to luminosities obtained in simulations with neutrino transport for non-rotating stars.

3

Conclusions and perspectives

Conclusions

• first step toward a realitistic study of the time evolution of rotating PNS;

• results from core collapses simulations taken into account;

• several effects found in the previous studies linked to unrealistic thermodynamical or rotational profiles;

• depending on the angular momentum at birth (and then efficiency of magnetic braking in iron cores): possible gravitational waves driven instabilities

$\longrightarrow$ possible gravitational burst after neutrinos peak.

Perspectives

- transport of angular momentum, neutrinos and energy between layers $\longrightarrow$ starting collaboration with Silvano Bonazzola (LUTH, Meudon) to study this with spectral method [time evolution, see Villain & Bonazzola (2002)];

- a lot of physical phenomena to add: magnetic field, convection, etc...

- situation completely different in which stationnary approximation would not be valid: merging of binary NS...