```
> restart;with(linalg);
Warning, new definition for norm
Warning, new definition for trace
```

[*BlockDiagonal, GramSchmidt, JordanBlock, LUdecomp, QRdecomp, Wronskian, addcol, addrow, adj, adjoint, angle, augment, backsub, band, basis, bezout, blockmatrix, charmat, charpoly, cholesky, col, coldim, colspace, colspan, companion, concat, cond, copyinto, crossprod, curl, definite, delcols, delrows, det, diag, diverge, dotprod, eigenvals, eigenvalues, eigenvectors, eigenvects, entermatrix, equal, exponential, extend, ffgausselim, fibonacci, forwardsub, frobenius, gausselim, gaussjord, geneqns, genmatrix, grad, hadamard, hermite, hessian, hilbert, htranspose, ihermite, indexfunc, innerprod, intbasis, inverse, ismith, issimilar, iszero, jacobian, jordan, kernel, laplacian, leastsqrs, linsolve, matadd, matrix, minor, minpoly, mulcol, mulrow, multiply, norm, normalize, nullspace, orthog, permanent, pivot, potential, randmatrix, randvector, rank, ratform, row, rowdim, rowspace, rowspan, rref, scalarmul, singularvals, smith, stack, submatrix, subvector, sumbasis, swapcol, swaprow, sylvester, toeplitz, trace, transpose, vandermonde, vecpotent, vectdim, vector, wronskian*]

```
> v:=vector(5,[1,2,8,-3,5]);
```

$$v := [1, 2, 8, -3, 5]$$

```
> vectdim(v);v[6];
```

$$5$$

```
Error, 1st index, 6, larger than upper array bound 5
> v:=vector(5,x->x^2-1); f := x->x^3+3;v:=vector(5,f);# on peut
  dfinir des vecteurs  l'aide de fonctions
```

$$v := [0, 3, 8, 15, 24]$$

$$f := x \to x^3 + 3$$

$$v := [4, 11, 30, 67, 128]$$

```
> v:=vector(4,x^2);# attention pas d'expressions
```

$$v := [\mathrm{x}(1)^2, \mathrm{x}(2)^2, \mathrm{x}(3)^2, \mathrm{x}(4)^2]$$

```
> v:=vector(n,f);#ni de dimension flottante
Error, (in vector) invalid arguments
> f := x->x^3+3;v:=vector(5,f);;v[3]:=12; v;eval(v);# on peut
  toujours modifier une composante
```

$$f := x \to x^3 + 3$$

$$v := [4, 11, 30, 67, 128]$$

$$v_3 := 12$$

$$v$$

$$[4, 11, 12, 67, 128]$$

```
> A:=matrix(4,4,[1,alpha*2+3,alpha^2 *2+4, 3*alpha^3
  +5,1,beta*2+3,beta^2 *2+4, 3*beta^3 +5,
  1,delta*2+3,delta^2 *2+4, 3*delta^3 +5,
  1,gamma*2+3,gamma^2 *2+4, 3*gamma^3 +5]);
```

$$A := \begin{bmatrix} 1 & 2\alpha+3 & 2\alpha^2+4 & 3\alpha^3+5 \\ 1 & 2\beta+3 & 2\beta^2+4 & 3\beta^3+5 \\ 1 & 2\delta+3 & 2\delta^2+4 & 3\delta^3+5 \\ 1 & 2\gamma+3 & 2\gamma^2+4 & 3\gamma^3+5 \end{bmatrix}$$

Evidemment on peut faire plus synthtique !

```
> AA:=matrix(4,4,[seq([1,2*x+3,2*x^2+4,3*x^3+5],x=[alpha,beta,gamma,delta])]);
```

$$AA := \begin{bmatrix} 1 & 2\alpha+3 & 2\alpha^2+4 & 3\alpha^3+5 \\ 1 & 2\beta+3 & 2\beta^2+4 & 3\beta^3+5 \\ 1 & 2\gamma+3 & 2\gamma^2+4 & 3\gamma^3+5 \\ 1 & 2\delta+3 & 2\delta^2+4 & 3\delta^3+5 \end{bmatrix}$$

```
> B:=matrix([[1,2,3],[2,3,4]]);# une matrice est une liste de liste (toujours en lignes)
```

$$B := \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \end{bmatrix}$$

```
> C:=matrix(3,3,(i,j)-> abs(i-j));# ou avec une fonction comme pour les vecteurs
```

$$C := \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & 0 \end{bmatrix}$$

```
> v:=vector(4,x->alpha*x^2);
```

$$v := [\alpha, 4\,\alpha, 9\,\alpha, 16\,\alpha]$$

```
> alpha:= 2;A; eval(v);eval(A);# pour les matrices comme pour les vecteurs attention l'valuation n'est pas fait composante composante
```

$$\alpha := 2$$

$$A$$

$$[\alpha, 4\,\alpha, 9\,\alpha, 16\,\alpha]$$

$$\begin{bmatrix} 1 & 2\alpha+3 & 2\alpha^2+4 & 3\alpha^3+5 \\ 1 & 2\beta+3 & 2\beta^2+4 & 3\beta^3+5 \\ 1 & 2\delta+3 & 2\delta^2+4 & 3\delta^3+5 \\ 1 & 2\gamma+3 & 2\gamma^2+4 & 3\gamma^3+5 \end{bmatrix}$$

```
> map(eval,A);map(eval,v); # mais heureusement il y a la fonction map ....
```

$$\begin{bmatrix} 1 & 7 & 12 & 29 \\ 1 & 2\beta+3 & 2\beta^2+4 & 3\beta^3+5 \\ 1 & 2\delta+3 & 2\delta^2+4 & 3\delta^3+5 \\ 1 & 2\gamma+3 & 2\gamma^2+4 & 3\gamma^3+5 \end{bmatrix}$$

$$[2, 8, 18, 32]$$

Pour rentrer le systme d'quation de l'exercice 1

```
> Eq:=[(m-1)*x+m*y+z = 1,m*x +2*y+3*z = 3, (m+1)*x+m*y+(m-1)*z =
  m-1];
```

$$Eq := [(m-1)\,x+m\,y+z=1,\ m\,x+2\,y+3\,z=3,\ (m+1)\,x+m\,y+(m-1)\,z=m-1]$$

```
> FF:=genmatrix(Eq,[x,y,z]);# Maple sait gnrer la matrice d'un
  systme que l'on a  rsoudre  condition de lui prciser les
  variables
```

$$FF := \begin{bmatrix} m-1 & m & 1 \\ m & 2 & 3 \\ m+1 & m & m-1 \end{bmatrix}$$

```
> F:=genmatrix(Eq,[x,y,z],flag);# gnre mme la matrice avec le
  second membre (trs intressant pour rsoudre des problmes de
  concourrance)
```

$$F := \begin{bmatrix} m-1 & m & 1 & 1 \\ m & 2 & 3 & 3 \\ m+1 & m & m-1 & m-1 \end{bmatrix}$$

```
> G1:=submatrix(F,1..3,2..4);# on peut extraire des sous matrices
  avec cette instruction qui garde des blocs contigs
```

$$G1 := \begin{bmatrix} m & 1 & 1 \\ 2 & 3 & 3 \\ m & m-1 & m-1 \end{bmatrix}$$

```
> G2:= submatrix(F,2..3,[1,2,4]);# ou avec les listes qui
  permettent alors un rarrangement on peut donc sans difficult
  calculer les solutions du systme Eq
```

$$G2 := \begin{bmatrix} m & 2 & 3 \\ m+1 & m & m-1 \end{bmatrix}$$

```
> m:=3; Eq; t:=det(F);t:=det(FF); y1:=det(G1)/t;
```

$$m := 3$$

$$[2\,x+3\,y+z=1,\ 3\,x+2\,y+3\,z=3,\ 4\,x+3\,y+2\,z=2]$$

```
Error, (in det) expecting a square matrix
```

$$t := 9$$

$$y1 := 0$$

Pour rsoudre le systme, on peut utiliser la fonction `solve` dj rencontre mais elle n'accepte que les ensembles d'quations

```
> solve(Eq,{x,y,z});Eq:={2*x+3*y+z = 1, 3*x+2*y+3*z = 3,
  4*x+3*y+2*z = 2 };solve(Eq,{x,y,z});
Error, (in solve) invalid arguments
```

$$Eq := \{4\,x+3\,y+2\,z=2,\ 3\,x+2\,y+3\,z=3,\ 2\,x+3\,y+z=1\}$$

$$\{y=0,\ x=0,\ z=1\}$$

```
> m:='m';Eq:={(m-1)*x+m*y+z = 1,m*x +2*y+3*z = 3,
  (m+1)*x+m*y+(m-1)*z = m-1};solve(Eq,{x,y,z});
```

$$m := m$$

$$Eq := \{(m+1)\,x+m\,y+(m-1)\,z=m-1,\ m\,x+2\,y+3\,z=3,\ (m-1)\,x+m\,y+z=1\}$$

$$\{y=0,\ x=0,\ z=1\}$$

```
> FF:=submatrix(F,1..3,1..3); b:=col(F,4); # b recoit la quatrime
  colonne
```

$$FF := \begin{bmatrix} m-1 & m & 1 \\ m & 2 & 3 \\ m+1 & m & m-1 \end{bmatrix}$$

$$b := [1,\ 3,\ m-1]$$

```
> linsolve(FF,b); # rsoud FF X = b attention m est ici affect
```

$$[0,\ 0,\ 1]$$

```
> m:='m'; FF:=submatrix(F,1..3,1..3); b:=col(F,4);
```

$$m := m$$

$$FF := \begin{bmatrix} m-1 & m & 1 \\ m & 2 & 3 \\ m+1 & m & m-1 \end{bmatrix}$$

$$b := [1,\ 3,\ m-1]$$

```
> linsolve(FF,b);
```

$$[0,\ 0,\ 1]$$

```
> alpha:='alpha'; evalm(A);A;
>
```

$$\alpha := \alpha$$

$$\begin{bmatrix} 1 & 2\,\alpha+3 & 2\,\alpha^2+4 & 3\,\alpha^3+5 \\ 1 & 2\,\beta+3 & 2\,\beta^2+4 & 3\,\beta^3+5 \\ 1 & 2\,\delta+3 & 2\,\delta^2+4 & 3\,\delta^3+5 \\ 1 & 2\,\gamma+3 & 2\,\gamma^2+4 & 3\,\gamma^3+5 \end{bmatrix}$$

$$A$$

```
> d:=det(A);
```

$$d := 12\,\gamma\alpha^3\,\delta^2 - 12\,\delta\,\alpha^3\,\gamma^2 - 12\,\gamma\alpha^2\,\delta^3 + 12\,\alpha\,\delta^3\,\gamma^2 - 12\,\alpha\,\delta^2\,\gamma^3 - 12\,\beta\,\alpha^3\,\delta^2 + 12\,\beta\,\alpha^2\,\delta^3$$
$$+ 12\,\delta\,\beta^3\,\gamma^2 + 12\,\delta\,\alpha^3\,\beta^2 + 12\,\beta\,\delta^2\,\gamma^3 - 12\,\gamma\beta^3\,\delta^2 - 12\,\beta\,\delta^3\,\gamma^2 + 12\,\gamma\beta^2\,\delta^3 - 12\,\delta\,\beta^2\,\gamma^3$$
$$- 12\,\delta\,\alpha^2\,\beta^3 + 12\,\delta\,\alpha^2\,\gamma^3 - 12\,\alpha\,\beta^2\,\delta^3 + 12\,\alpha\,\beta^3\,\delta^2 - 12\,\alpha\,\beta^3\,\gamma^2 - 12\,\beta\,\alpha^2\,\gamma^3 - 12\,\gamma\alpha^3\,\beta^2$$
$$+ 12\,\gamma\alpha^2\,\beta^3 + 12\,\beta\,\alpha^3\,\gamma^2 + 12\,\alpha\,\beta^2\,\gamma^3$$

```
> l:=factor(d);
```

$$l := -12\,(-\gamma+\alpha)\,(-\gamma+\beta)\,(\beta-\alpha)\,(-\gamma+\delta)\,(\delta-\alpha)\,(\delta-\beta)$$

```
> det(FF);P:=unapply(det(FF),m);
```

$$4\,m^2 - m^3$$

$$P := m \to 4\,m^2 - m^3$$

```
> H1:=matrix([[2/6,1/6],[1/6,2/6]]);H:= matrix([[H1,H1],[H1,H1]]);
```

$$H1 := \begin{bmatrix} \frac{1}{3} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{3} \end{bmatrix}$$

$$H := \begin{bmatrix} H1 & H1 \\ H1 & H1 \end{bmatrix}$$

Il existe une commande Maple pour crer des matrices par blocs :

```
> HH:=blockmatrix(2,2,H1,H1,H1,H1);
```

$$HH := \begin{bmatrix} \frac{1}{3} & \frac{1}{6} & \frac{1}{3} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{3} & \frac{1}{6} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{6} & \frac{1}{3} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{3} & \frac{1}{6} & \frac{1}{3} \end{bmatrix}$$

```
> evalm(H);
```

$$\begin{bmatrix} H1 & H1 \\ H1 & H1 \end{bmatrix}$$

Il vaut donc mieux utiliser blockmatrix mais l'obstination de Maple va nous permettre de voir ce qui se passe quand on lve au carr une telle matrice

```
> HCARRE:=H^2;evalm(HCARRE); HHCAREE:=evalm(HH^2);
```

$$HCARRE := H^2$$

$$\begin{bmatrix} 2\,H1^2 & 2\,H1^2 \\ 2\,H1^2 & 2\,H1^2 \end{bmatrix}$$

$$HHCAREE := \begin{bmatrix} \frac{5}{18} & \frac{2}{9} & \frac{5}{18} & \frac{2}{9} \\ \frac{2}{9} & \frac{5}{18} & \frac{2}{9} & \frac{5}{18} \\ \frac{5}{18} & \frac{2}{9} & \frac{5}{18} & \frac{2}{9} \\ \frac{2}{9} & \frac{5}{18} & \frac{2}{9} & \frac{5}{18} \end{bmatrix}$$

```
> f:=n-> evalm(HH^n);
```

$$f := n \rightarrow \mathrm{evalm}(HH^n)$$

```
> f(2);
```

$$\begin{bmatrix} \frac{5}{18} & \frac{2}{9} & \frac{5}{18} & \frac{2}{9} \\ \frac{2}{9} & \frac{5}{18} & \frac{2}{9} & \frac{5}{18} \\ \frac{5}{18} & \frac{2}{9} & \frac{5}{18} & \frac{2}{9} \\ \frac{2}{9} & \frac{5}{18} & \frac{2}{9} & \frac{5}{18} \end{bmatrix}$$

On aurait d y penser !

```
> f(30);
```

$$\begin{bmatrix} \frac{102945566047325}{411782264189298} & \frac{51472783023662}{205891132094649} & \frac{102945566047325}{411782264189298} & \frac{51472783023662}{205891132094649} \\ \frac{51472783023662}{205891132094649} & \frac{102945566047325}{411782264189298} & \frac{51472783023662}{205891132094649} & \frac{102945566047325}{411782264189298} \\ \frac{102945566047325}{411782264189298} & \frac{51472783023662}{205891132094649} & \frac{102945566047325}{411782264189298} & \frac{51472783023662}{205891132094649} \\ \frac{51472783023662}{205891132094649} & \frac{102945566047325}{411782264189298} & \frac{51472783023662}{205891132094649} & \frac{102945566047325}{411782264189298} \end{bmatrix}$$

```
> map(evalf,");
```

$$\begin{bmatrix} .2500000000 & .2500000000 & .2500000000 & .2500000000 \\ .2500000000 & .2500000000 & .2500000000 & .2500000000 \\ .2500000000 & .2500000000 & .2500000000 & .2500000000 \\ .2500000000 & .2500000000 & .2500000000 & .2500000000 \end{bmatrix}$$

```
> Digits:= 8;seq(map(evalf,f(i)), i=[3,9,15,27]);
```

$$Digits := 8$$

$$\begin{bmatrix} .25925926 & .24074074 & .25925926 & .24074074 \\ .24074074 & .25925926 & .24074074 & .25925926 \\ .25925926 & .24074074 & .25925926 & .24074074 \\ .24074074 & .25925926 & .24074074 & .25925926 \end{bmatrix},$$

$$\begin{bmatrix} .25001270 & .24998730 & .25001270 & .24998730 \\ .24998730 & .25001270 & .24998730 & .25001270 \\ .25001270 & .24998730 & .25001270 & .24998730 \\ .24998730 & .25001270 & .24998730 & .25001270 \end{bmatrix},$$

$$\begin{bmatrix} .25000002 & .24999998 & .25000002 & .24999998 \\ .24999998 & .25000002 & .24999998 & .25000002 \\ .25000002 & .24999998 & .25000002 & .24999998 \\ .24999998 & .25000002 & .24999998 & .25000002 \end{bmatrix},$$

$$\begin{bmatrix} .25000000 & .25000000 & .25000000 & .25000000 \\ .25000000 & .25000000 & .25000000 & .25000000 \\ .25000000 & .25000000 & .25000000 & .25000000 \\ .25000000 & .25000000 & .25000000 & .25000000 \end{bmatrix}$$

```
> tH:=transpose(HH);#on a transpos mais cela ne se voit pas !
```

$$tH := \begin{bmatrix} \frac{1}{3} & \frac{1}{6} & \frac{1}{3} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{3} & \frac{1}{6} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{6} & \frac{1}{3} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{3} & \frac{1}{6} & \frac{1}{3} \end{bmatrix}$$

```
> spectre:=eigenvalues(tH);
```

$$spectre := 0, 0, \frac{1}{3}, 1$$

Pour des valeurs numriques on peut aussi faire:

```
>  spectre2:=evalf(Eigenvals(tH, vecs));
```

$$spectre2 := [.30897999\ 10^{-9}, .11855987\ 10^{-7}, .33333332, 1.0000000]$$

```
> p:=print(vecs);# vecs contient les vecteurs propres approchs
```

$$\begin{bmatrix} .65328148 & -.27059802 & -.49999999 & .50000001 \\ -.27059806 & -.65328143 & .50000001 & .50000001 \\ -.65328143 & .27059808 & -.50000004 & .50000002 \\ .27059803 & .65328145 & .50000002 & .49999999 \end{bmatrix}$$

$$p :=$$

```
> eigenvectors(tH);# le calcul est fait ici de faon symbolique car
  tous les coefficient de la matrice sont entiers
```

$$[0, 2, \{[-1, 0, 1, 0], [0, -1, 0, 1]\}], \left[\frac{1}{3}, 1, \{[-1, 1, -1, 1]\}\right], [1, 1, \{[1, 1, 1, 1]\}]$$

```
> eigenvalues(A):#vrifier ce que Maple sort !
> A2 := matrix(3,3,[0.3,-0.4,0,0.4,0.3,0,0,0,1]);eigenvalues(A2);
```

$$A2 := \begin{bmatrix} .3 & -.4 & 0 \\ .4 & .3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$.30000000 + .40000000\, I,\ .30000000 - .40000000\, I,\ 1.$$

```
> band([b,a,b], n);n:=3;A3ab:=band([b,a,b], n);
Error, (in band) wrong number or type of arguments
```

$$n := 3$$

$$A3ab := \begin{bmatrix} a & b & 0 \\ b & a & b \\ 0 & b & a \end{bmatrix}$$

```
> charmat(A3ab,x);charpoly(A3ab,x);#comme leur nom l'indique
```

$$\begin{bmatrix} x - a & -b & 0 \\ -b & x - a & -b \\ 0 & -b & x - a \end{bmatrix}$$

$$x^3 - 3\, x^2\, a + 3\, x\, a^2 - 2\, x\, b^2 - a^3 + 2\, a\, b^2$$

```
> eigenvalues(A3ab);
```

$$a,\ a + \sqrt{2}\, b,\ a - \sqrt{2}\, b$$

```
>
```