CONFERENCE SPECTRAL THEORY AND GEOMETRY, 28TH SEPT. 2020 - 02 OCT 2020 - CHALÈS

1. Programme

Monday 28	Tuesday 29	Wednesday 30	Thursday 1	Friday 6
10.00 Welcome coffee	**9.30** C. Laurent	**9.30** A. Koenig	**9.30** C. Fermanian	**9.30** T. Ourmières-Bonafos
	10.30 Coffee break	**10.30** Coffee break	**10.30** Coffee break	**10.30** Coffee break
11.00 G. Rivière	**11.00** To be precised	**11.00** R. Pétrides	**11.00** E. Le Masson	**11.00** C. Sun
12.00 Lunch	**12.00** Lunch	**12.00** Lunch	**12.00** Lunch	**12.00** Lunch
14.15 E. Trélat	**14.15** Y. Colin de Verdière (video)	**14.00** Excursion	**14.15** D. Vicente	**14.15**
15.15 Coffee Break	**15.15** Coffee Break	in	**15.15** Coffee Break	**15.15** Conf. ending
15.45F. Macià	**15.45** M. Léautaud	Chambord	**15.45** C. Letrouit	
19.00 Dinner	**19.00** Dinner	**19.00** Dinner	**19.00** Gala Dinner	

2. Liste des exposés

- **Y. Colin de Verdières :** *Essential self-adjointness of differential operators on closed manifolds.*

  This is joint work with Corentin Le Bihan (ENS Lyon). A classical result of Gaffney says that a Riemannian laplacian on a complete Riemannian manifold is essentially self-adjoint (ESA). Of course this applies in particular to closed manifolds. It holds true even for sR Laplacians. In this lecture, I will study the case of non sub-elliptic operators. A basic example for geometers is the Laplacian of a Lorentzian metric. Generically in dimension 2, such a Laplacian is not essentially self-adjoint. I will also discuss the case of higher dimensions which is related to a recent work in collaboration with Laure Saint-Raymond on a spectral problem coming from the study of internal waves. A general conjecture will appear relating ESA to classical completeness of the Hamiltonian vector field of the symbol.

- **C. Fermanian :** *Observability for the Schrdinger equation on quotient manifolds of Heisenberg type.*

In this talk, we discuss a recent result obtained in collaboration with Cyril Letrouit that states geometric conditions for observability in a sub-elliptic context. From Harmonic analysis point of view, the operator that we consider is the sub-Laplacian associated with a 2-step stratified Lie group of Heisenberg type and we work on a quotient manifold. Our arguments use a semi-classical approach based on representation theory and the introduction of related wave packets that we will explain.

- **A. Koenig :** *Control of the fractional heat equation and related equations.*

  We know since 1995 and the work of Lebeau-Robbiano and Fursikov-Immanuvilov that the heat equation is null-controllable in arbitrarily small time. We will discuss the case of the fractional heat equation and equations that looks like the fractional heat equation. These equations are useful in the study of some degenerate parabolic equation, such as the parabolic Grushin equation and some Kolmogorov-type equations, that behave in some regime like the fractional heat equation. We will show in particular how some (relatively) simple tools from complex analysis can be used to exhibit necessary geometric control conditions for the control of the parabolic Grushin equation.

- **C. Laurent :** *On the control of coupled wave equations.*

  In this talk, we will report on a recent article with Yan Cui and Zhiqiang Wang about the control of systems of wave equations coupled by lower orders terms. We will describe the link with an ODE system along the Hamiltonian flow.

- **E. Le Masson :** *Eigenfunctions of the Laplacian on random hyperbolic surfaces of large genus.*

- **M. Léautaud :** *On uniform observability of gradient flows in the vanishing viscosity limit.*

  We consider a transport equation by a gradient vector field with a small viscous perturbation $-\varepsilon\Delta_g$. We study uniform observability properties in the (singular) vanishing viscosity limit $\varepsilon \to 0^+$, that is, the possibility of having a uniformly bounded observation constant. We prove with a series of examples that in general, the minimal time for uniform observability may be much larger than the minimal time needed for the observability of the limit equation $\varepsilon = 0$. We also prove that the two minimal times coincide for positive solutions. In dimension one, we finally compute more precise bounds on the minimal time for uniform observability. This is a joint work with Camille Laurent.

- **C. Letrouit :** *Quantum limits of products of Heisenberg manifolds.*

In Riemannian geometry, the distribution on the manifold of high-frequency eigenfunctions of the Laplace-Beltrami operator heavily depends on the properties of the geodesic flow: if it is ergodic, nearly all eigenfunctions become equidistributed in the high-frequency limit, whereas eigenfunctions of completely integrable systems, due to the high multiplicity of some eigenvalues, may present more complicated patterns. In this talk, we deal with the same problem in the more general framework of sub-Riemannian geometry. Given $m \in \mathbb{N}$, we consider the sub-Laplacian $\sum_{j=1}^{m} \partial_{x_j}^2 + (\partial_{y_j} - x_j \partial_{z_j})^2$, which is the natural sub-Laplacian on products of compact quotients of the 3D Heisenberg group. The associated geodesic flow is completely integrable, and the study of the Quantum Limits, which characterize possible limits of high-frequency eigenfunctions, reveals a very rich structure, in which an infinite number of flows comes into play.

- **F. Macia :** *Modes, quasi-modes and the perturbed harmonic oscillator.*

  I will present some recent result on concentration properties on phase-space for eigenstates of small perturbations of Quantum Completely Integrable Schrdinger equations, with particular emphasis on the semiclassical Quantum Harmonic Oscillator. The effect of the perturbation turns out to depend on the arithmetic properties of the frequencies of the oscillator. We will illustrate this type of effect by comparing the sets of semiclassical measures of sequences of eigenfunctions in the perturbed and unperturbed cases.This is joint work with Vctor Arnaiz (Orsay).

- **T. Ourmières-Bonafos :** *A non-linear variational characterization for Dirac eigenvalues in bounded domains. Application to spectral geometric inequalities.*

  In this talk, I will discuss a non-linear variational characterization of the principal eigenvalue of the Dirac operator with infinite mass boundary condition in a smooth bounded and simply connected plane domain. This variational characterization allows both for numerical and theoretical results. In particular, I will explain how we use this variational characterization to compare the principal eigenvalue of any such domain with the one of the unit disk in terms of simple geometric quantities. This is joint work with: P. Antunes, R.D. Benguria, V. Lotoreichik

- **R. Petrides :** *Free boundary minimal surfaces of any topological type in Euclidean balls via shape optimization.*

  Maximal metrics for the isoperimetric problem for Steklov eigenvalues, arise as induced metrics of free boundary minimal surfaces in Euclidean balls. Then it is natural to perform a variational method on Steklov eigenvalues in order to build new minimal surfaces, as stated in previous works by Fraser-Schoen and Petrides. The program of building free boundary minimal surfaces into Euclidean balls of any topological type is now completed by this method. It is a consequence of new gap results on first eigenvalues with respect to the topology in a recent joint work with Matthiesen. I will give some idea of the glueing construction and the asymptotic analysis behind these results.

- **G. Rivière :** *Poincar series and linking of Legendrian knots.*

  On a compact surface of variable negative curvature, I will explain that the Poincar series associated to the geodesic arcs joining two given points has a meromorphic continuation to the whole complex plane. Moreover, its value at 0 is rationnal and it can be expressed in terms of the genus of the surface by interpreting it in terms of the linking of two Legendrian knots. If time permits, I will explain how this result extends when one considers geodesic arcs orthogonal to two fixed closed geodesics. This is a joint work with N.V. Dang (Lyon).

- **C. Sun :** *Observability and Resolvent estimate for the Grushin-type operators.*
  In this talk we consider the observability property for some evolution equations associated with the Grushin operator $\Delta_\gamma = \partial_x^2 + |x|^{2\gamma}\partial_y^2$ on a finite cylinder $M = (-1,1)_x \times \mathbb{T}_y$. The observation region is a horizontal strip ω. First, we give a resolvent estimate which excludes the concentration in ω of certain quasi-modes. Consequently, we obtain the observability for the Schrödinger type equation $i\partial_t u - (-\Delta_\gamma)^s u = 0$ when $s \geq \frac{\gamma+1}{2}$ and the energy decay for the damped wave equation $\partial_t^2 u - \Delta_\gamma u + \mathbf{1}_\omega \partial_t u = 0$. Due to the concentration property of the eigenfunctions of the generalized harmonic oscillator, the relation $s \geq \frac{\gamma+1}{2}$ is also necessary for the observability of the Schrödinger equation. This is a joint work with Cyril Letrouit.

- **E. Trélat :** *Lois de Weyl en géométrie sous-Riemannienne.*

  Dans cet exposé, je montrerai comment calculer l'asymptotique en temps petit de la loi locale de Weyl pour un Laplacien sous-Riemannien, c'est-à-dire un Laplacien qui est une somme de carrés de champs de vecteurs vérifiant l'hypothèse de Hormander. Les cas singuliers (i.e., non quiréguliers) sont les plus délicats, comme par exemple Baouendi-Grushin ou Martinet.

- **D. Vicente :** *A perfect reconstruction property for PDE-constrained total-variation minimization.*

  We study the recovery of piecewise constant functions of finite bounded variation which are constrained to satisfy a linear partial differential equation with unknown boundary conditions. We prove an exact recovery result up to a global constant under a mild geo- metric assumption on the jump set of the function to reconstruct. For that, we establish some theoretical results about BV spaces which consist in the generalization of the notion of wavefront in this setting. To demonstrate the practical relevance of these results, we also discuss the application to quantitative susceptibility mapping (QSM), a recently established magnetic resonance imaging (MRI) technique which aims at providing the spatial susceptibility distribution of tissues inside the human body. This is a joint work with Kristian Bredies (University of Graz, Austria).